Introduction to GAMS

An Example of a Transportation Problem

Example – A Transportation Problem

 maximizing  \[ z = \sum_{i} \sum_{j} c_{ij}x_{ij} \]

subject to:

\[ \sum_{i} x_{ij} \geq b_j \quad j = 1, \ldots, m \]

\[ \sum_{j} x_{ij} \leq a_i \quad i = 1, \ldots, n \]
Video Goes Here
Sets
   1 canning plants / seattle, san-diego /
   2 markets / new-york, chicago, topeka / ;

Parameters
   m(i) capacity of plant i in cases
      / seattle 350 san-diego 400 / ;
   b(j) demand at market j in cases
      / new-york 325 chicago 300 topeka 275 / ;
Table d(i,j) distance in thousands of miles
         new-york    chicago    topeka
   seattle      2.5        1.7       1.8
   san-diego    2.5        1.8       1.6 ;

Scalar f freight in dollars per case per thousand miles /90/ ;
Parameter c(i,j) transport cost in thousands of dollars per case ;

\[
c(i,j) = f \times d(i,j) / 1000 ;
\]

Variables
   x(i,j) shipment quantities in cases
   z total transportation costs in thousands of dollars ;

Positive Variable x ;

Equations
   cost define objective function
   supply(i) observe supply limit at plant i
   demand(j) satisfy demand at market j ;

   cost ..  \[ z = \text{e} = \sum_{i,j} c(i,j) \times x(i,j) ; \]
   supply(i) ..  \[ \sum_{j} x(i,j) = a(i) ; \]
   demand(j) ..  \[ \sum_{i} x(i,j) = b(j) ; \]

Model transport /all/ ;
Solve transport using ip minimizing z ;
Display x.l, x.m ;
This problem finds a least cost shipping schedule that meets requirements at markets and suppliers at factories.

References:
Dantzig, G B., Linear Programming and Extensions
Princeton University Press, Princeton, New Jersey, 1963,
Chapter 3-1.

This formulation is described in detail in Chapter 2
(A. E. S. Cooke, D. Kendrick and A. Meeraus, The Scientific Press,
Redwood City, California, 1981.)

The line numbers will not match those in the book because of these comments.

Sets
1. canning plants / seattle, san-diego /
2. j. markets / new-york, chicago, topeka / ;

Parameters
31. a(i) capacity of plant i in cases
32. / seattle 500
33. / san-diego 600 /
34. b(j) demand at market j in cases
35. / new-york 350
36. / chicago 300
37. / topeka 275 / ;

Table d(i,j) distance in thousands of miles
38. new-york chicago topeka
39. seattle 2.5 1.7 1.0
40. san-diego 2.5 1.0 1.2

Scalar f freight in dollars per case per thousand miles /90/ ;

Parameter c(i,j) transport cost in thousands of dollars per case ;
46. c(i,j) = f * d(i,j) / 900 ;
Variables

x(i,j) shipment quantities in cases
z total transportation costs in thousands of dollars

Positive Variable x;

Equations
cost define objective function
supply(i) observe supply limit at plant i

A Transportation Problem [TRANSOPT,SEQ=1]

\[
\text{demand}(j) \text{ satisfy demand at market } j ;
\]

\[
\text{cost} \quad z = \sum_{i,j} c(i,j) \cdot x(i,j) ;
\]

\[
\text{supply}(i) \quad \sum_{j} x(i,j) = b(i) ;
\]

\[
\text{demand}(j) \quad \sum_{i} x(i,j) = d(j) ;
\]

Model transport /all/ ;
Solve transport using ip minimize z ;

Display x.l, x.m ;

--- COST define objective function

COST.. 0.225*x(seattle,new-york) + 0.153*x(seattle,chicago)
        - 0.162*x(seattle,tokyo) - 0.225*x(san-diego,new-york)
        = 0.162*x(san-diego,chicago) + 0.128*x(san-diego,tokyo) + z = 0 ;

---- SUPPLY observe supply limit at plant i

SUPPLY(seattle).. x(seattle,new-york) + x(seattle,chicago)
---- SUPPLY
"L" observe supply limit at plant i

SUPPLY(seattle).. X(seattle,new-york) + X(seattle,chicago) + X(seattle, topeka) =L= 350 ; (LHS = 0)

SUPPLY(san-diego).. X(san-diego,new-york) + X(san-diego,chicago) + X(san-diego, topeka) =L= 600 ; (LHS = 0)

---- DEMAND
"M" satisfy demand at market j

DEMAND(new-york).. X(seattle,new-york) + X(san-diego,new-york) =G= 325 ;
(LHS = 0, INFES = 325 ***)

DEMAND(chicago).. X(seattle,chicago) + X(san-diego,chicago) =G= 300 ;
(LHS = 0, INFES = 300 ***)

DEMAND(topeka).. X(seattle,topeka) + X(san-diego,topeka) =G= 275 ;
(LHS = 0, INFES = 275 ***)

---- X
shipment quantities in cases

X(seattle,new-york)
.LO .L .UF = 0 0 +INF
=0.325 COST
1 SUPPLY(seattle)
1 DEMAND(new-york)

X(seattle,chicago)
.LO .L .UF = 0 0 +INF
=0.153 COST
1 SUPPLY(seattle)
1 DEMAND(chicago)

X(seattle,topeka)
.LO .L .UF = 0 0 +INF
X(seattle, topeka)
(.LO, .L, .UP = 0, 0, +INF)
-0.142 COST
1 SUPPLY(seattle)
1 DEMAND(topeka)

REMAINING 3 ENTRIES SKIPPED

---

TOTAL TRANSPORTATION COSTS IN THOUSANDS OF DOLLARS

2

(.LO, .L, .UP = -INF, 0, +INF)
1 COST

MODEL STATISTICS

BLOCKS OF EQUATIONS 3 SINGLE EQUATIONS 6

EXECUTION TIME = 0.640 SECONDS 1.4 Kb WIN-18-096

SOLVE SUMMARY

MODEL TRANSPORT OBJECTIVE Z
MODEL TRANSPORT  OBJECTIVE 2
TYPE LP  DIRECTION MINIMIZE
SOLVER MINOS  FROM LINE 70

***** SOLVER STATUS 1 NORMAL COMPLETION
***** MODEL STATUS 1 OPTIMAL
***** OBJECTIVE VALUE 153.6750

RESOURCE USAGE, LIMIT 0.551  10000.000
ITERATION COUNT, LIMIT 5  10000

MINOS  Feb 28, 1999 WIN.MS.18.0 105.094.036.WAT GAMS/MIOS 5.4

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and
P. E. Gill, W. Murray, M. A. Saunders and M. H. Wright
Systems Optimization Laboratory, Stanford University.

Work space allocated -- 0.04 Mb

-- OPTIMAL SOLUTION FOUND

---- EQU COST

<table>
<thead>
<tr>
<th>LOWER</th>
<th>LEVEL</th>
<th>UPPER</th>
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COST  define objective function

---- EQU SUPPLY  observe supply limit at plant i

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seattle  -INF  300.000  350.000  .
san-diego -INF  600.000  600.000  .

---- EQU DEMAND  satisfy demand at market j

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new-york  325.000  325.000  +INF  0.225
GAMS View of a Problem

\[
\begin{align*}
\text{optimize } & \quad z \\
\text{subject to :} & \quad z = f(x) \\
& \quad g_j \leq g_j(x) \leq \bar{g}_j \quad j = 1, \ldots, m \\
& \quad l_i \leq x_i \leq u_i \quad i = 1, \ldots, n
\end{align*}
\]

Bounded Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Lower Bd.</th>
<th>Upper Bd.</th>
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<tbody>
<tr>
<td>( \leq )</td>
<td>(-\infty)</td>
<td>(b)</td>
</tr>
<tr>
<td>( \geq )</td>
<td>(b)</td>
<td>(+\infty)</td>
</tr>
<tr>
<td>( = )</td>
<td>(b)</td>
<td>(b)</td>
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</table>

Finite upper and lower bounds simultaneously can be accommodated
Shadow Prices/Penalty Costs

- Shadow prices are “marginals”

\[
\begin{align*}
\text{optimize} & \quad z \\
\text{subject to:} & \\
& z = f(x) \\
& g_j \leq g_j(x) \leq \bar{g}_j \quad j = 1, \ldots, m \\
& l_i \leq x_i \leq u_i \quad i = 1, \ldots, n
\end{align*}
\]
### EQU DEMAND

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<tr>
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<td>325,000</td>
<td>+INF</td>
</tr>
<tr>
<td>chicago</td>
<td>300,000</td>
<td>300,000</td>
<td>+INF</td>
</tr>
<tr>
<td>topeka</td>
<td>275,000</td>
<td>275,000</td>
<td>+INF</td>
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A Transportation Problem (TRANSPOST, SEQ=1)

### VAR X

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<td>new-york</td>
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<td>chicago</td>
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<tr>
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<td>+INF</td>
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<tr>
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<td>chicago</td>
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<tr>
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### VAR Z

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<tr>
<td>.</td>
<td>-INF</td>
<td>155.475</td>
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Z total transportation costs in thousands of dollars

### REPORT SUMMARY:

| 0 | SCANEOF |
| 0 | INFEASIBLE |
| 0 | UNBOUNDED |

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A Transportation Problem (TRANSPOST, SEQ=1)

### 72 VARIABLE X.L

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</table>

### 72 VARIABLE X.M

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