Dynamic, Deterministic Models

- Key to Correct Formulation of Dynamic Models: UNITS!
  - In addition to bushels, tons and kilograms, units now include when (i.e., a date)
    - Bushels sold in December
    - Kilograms of fertilizer applied in the first week of the rainy season
    - Warehouse capacity available today

Why is dynamic different?

- The other distinguishing feature of dynamic models is the one-way flow of commodities and value
  - Physical commodities flow forward through time – inventory storage
  - Value (shadow prices) flow backward through time (i.e., today’s value of a stock of a commodity or resource depends not only on value for use today but also on value for use in the future)
Types of Dynamic Models

The two principle types of dynamic models are

- Scheduling models -- focus on how to allocate fixed resources across time
- Inventory models -- focus on how to manage flows of resources across time

Scheduling Models

- With scheduling models, we typically have a number of fixed resources whose use must be scheduled over time
  - Land use (crop mix) may be scheduled over one or more crop years
  - Machinery use may be scheduled over planting and/or harvesting seasons
  - Utilization of livestock facilities may be scheduled
Scheduling Models (cont’d.)

- Warehouse/grain bin utilization may be scheduled over time
- Personnel usage (task assignment) may be scheduled
- Transportation facilities use may get scheduled (e.g., truck, trains, planes, depots, gates)
- Maintenance may get scheduled

Inventory Models

- Inventory Models -- The other big type of dynamic model
  - Focus is on managing flows of “commodities” over time
  - These commodities are storable over time (perhaps within limits due to perishability)
  - As with most flow systems, leakage (storage losses) may be a problem
Inventory Models (cont’d.)

Examples of inventory models include –

• Resource (renewable or nonrenewable) management (oil and other fossil fuels, soil, water, fish, etc.)

• Storage of commodities (timing/locations of supply, and timing/locations of demand in combination with differences in storage costs/quality create opportunities)

• Inputs to manufacturing processes must be available in a timely fashion to avoid production shortfalls and/or idling of productive resources

Inventory Example – A Grain Marketing System

Consider the example of the Zambian maize marketing system (Mwanaumo, 1994)

System features:

• Maize is produced at different levels of intensity in different parts of the country

• Maize is consumed at different rates in different parts of the country (different from prod. pattern)
Maize Marketing System (cont’d.)

- The government controls transport and marketing of maize around the country

- There are inter-regional shipping losses independent of source and destination

- The regional network is not directly, fully connected – shipments from Kabwe to Solwezi may not be possible, but shipment from Kabwe to Kitwe and then Kitwe to Solwezi are feasible

Maize Marketing System (cont’d.)

- Supply becomes available primarily in the second quarter of the marketing year with some additional supplies in the first and third quarters

- The maize supplied to the marketing system by farmers must be processed before delivery to consumers

- There are two processed products – breakfast and roller
Maize Marketing System (cont’d.)

- There are capacity restrictions in each region for storage and processing
- Storage costs vary by region
- Transportation costs between regions are linear in distance between the regions

This is an ideal case for an inventory model
- Multiple locations
- Multiple products
- Capacity limits

The goal will be to minimize the sum of
- Transportation costs
- Storage costs
- Processing costs
- Subject to satisfying consumer demand
Maize Marketing System (cont’d.)

Consider the region I, period T maize commodity balance constraint:

\[
\text{MAIZE}(I,T) \leq \text{SUPPLY}(I,T) + \text{STOREM}(I,T-1)(1-\text{SLOSS})
\]

\[
+ \sum(J) \text{SHIPM}(J,I,T)(1-\text{TLOSS}) = \text{G} = \sum(J) \text{SHIPM}(I,J,T) + \text{STOREM}(I,T)
\]

\[
+ \frac{1}{\text{YIELDR}} \text{MAKER}(I,T) + \frac{1}{\text{YIELDB}} \text{MAKEB}(I,T)
\]

This is a standard “sources at least as great as uses” constraint

---

Maize Marketing System (cont’d.)

The units of this constraint are metric tons of maize in region I, period (quarter) T

\text{SHIPM}(I,J,T) denotes shipments of maize from I to J in period T

\text{STOREM}(I,T) denotes storage of maize in region I from period T to period T+1

\text{MAKER}(I,T) denotes the number of metric tons of roller produced in region I, period T
Maize Marketing System (cont’d.)

- YELDR is the yield of roller (metric tons of roller produced per metric ton of maize)

- MAKEB(I,T) denotes the number of metric tons of breakfast produced in region I, period T

- YELDB is the yield of breakfast (metric tons of breakfast produced per metric ton of maize)

- SUPPLY(I,T) denotes the metric tons of maize coming into the system in region I, period T

Maize Marketing System (cont’d.)

- TLOSS denotes the shipping loss factor – for each ton of maize shipped from the source, (1-TLOSS) tons of maize arrive at the destination

- SLOSS denotes the storage loss factor – for each ton of maize stored in period T, (1-SLOSS) tons of maize are available in period T+1
Maize Marketing System (cont’d.)

- Now consider the contributions of these variables to the objective:

- \[
    \text{SUM}((I,J,T), \text{TCOSTM}(I,J) \cdot \text{SHIPM}(I,J,T)) \\
    + \text{SUM}((I,T), \text{SCOSTM}(I,T) \cdot \text{STOREM}(I,T)) \\
    + \text{SUM}((I,T), \text{MCOSTR}(I) \cdot \text{MAKER}(I,T)) \\
    + \text{SUM}((I,T), \text{MCOSTB}(I) \cdot \text{MAKEB}(I,T))
    \]

- These are the costs for shipping maize, storing maize, processing roller and breakfast.

---

Maize Marketing System (cont’d.)

- As we have seen before, the transportation variables (\(\text{SHIPM}(I,J,T)\)) link the value of maize across regions.

- The FOC for \(\text{SHIPM}(I,J,T)\) is:

- \[
    \text{TCOSTM}(I,J) + \text{MAIZE.M}(I,T) \\
    - \text{MAIZE.M}(J,T) \cdot (1 - \text{TLOSS}) \geq 0
    \]
Maize Marketing System (cont’d.)

- or

- \[ \text{MAIZE}.M(I,T) \geq \text{MAIZE}.M(J,T) \times (1 - \text{TLOSS}) - \text{TCOSTM}(I,J) \]

- Note that this relationship will be an equality if \( \text{SHIPM}(I,J,T) > 0 \) and the inequality may be strict otherwise

- This is an arbitrage condition that relates value of maize across regions

---

Maize Marketing System (cont’d.)

- The storage activities have a similar function, but they relate values across time periods
- Recall that storage is limited by capacity:

- \[ \text{STOREM}.\text{UP}(I,T) = \text{SCAPM}(I) \]

- The FOC for \( \text{STOREM}(I,T) \) is

- \[ \text{SCOSTM}(I) + \text{MAIZE}.M(I,T) - \text{MAIZE}.M(I,T+1) \times (1 - \text{SLOSS}) - \text{STOREM}.M(I,T) \geq 0 \]
Maize Marketing System (cont’d.)

or

\[
\text{MAIZE.M}(I,T) \geq \text{SCOSTM}(I) + \text{MAIZE.M}(I,T+1)*(1 - \text{SLOSS}) - \text{STOREM.M}(I,T)
\]

This will be an equality if 0<STOREM(I,T), but left- and right-hand sides may not be equal otherwise

This too is an arbitrage condition, but now it is across time periods within a region (Notice STOREM.M(I,T)<0)

Maize Marketing System (cont’d.)

To see the other relationships, we introduce the balance equations for processed products

For “roller”

\[
\text{ROLLER}(I,T) \leq \text{MAKER}(I,T)
\]

\[
+ \text{STORER}(I,T-1)*(1 - \text{SLOSS})
\]

\[
+ \text{SUM}(J,\text{SHIPR}(J,I,T))*(1 - \text{TLOSS})
\]

\[
= \text{G} = \text{SUM}(J,\text{SHIPR}(I,J,T)) + \text{STORER}(I,T)
\]

+ \text{DEMANDR}(I,T)
Maize Marketing System (cont’d.)

- and for “breakfast”

- \[ BFAST(I,T) \text{ .. MAKEB}(I,T) \]
  \[ + \text{STOREB}(I,T-1) \times (1 - \text{SLOSS}) \]
  \[ + \text{SUM}(J, \text{SHIPB}(J,I,T)) \times (1 - \text{TLOSS}) \]
  \[ \text{=G=} \text{SUM}(J, \text{SHIPB}(I,J,T)) \]
  \[ + \text{STOREB}(I,T) + \text{DEMANDB}(I,T) \]

- Again, these are “sources at least as great as uses” constraints

Maize Marketing System (cont’d.)

- The objective contributions for the new variables are:

  - \[ \text{SUM}((I,J,T), \text{TCOSTR}(I,J) \times \text{SHIPR}(I,J,T)) \]
  - \[ + \text{SUM}((I,T), \text{SCOSTR}(I) \times \text{STORER}(I,T)) \]
  - \[ + \text{SUM}((I,J,T), \text{TCOSTB}(I,J) \times \text{SHIPB}(I,J,T)) \]
  - \[ + \text{SUM}((I,T), \text{SCOSTB}(I) \times \text{STOREB}(I,T)) \]

- The price linkages across time are the same as for maize, but for the processed products
Maize Marketing System (cont’d.)

- The FOC for SHIPR(I,J,T) is:
  
  TCOSTR(I,J) + ROLLER.M(I,T) - ROLLER.M(J,T)*(1 - TLOSS) ≥ 0

- Or,

  ROLLER.M(I,T) ≥ ROLLER.M(J,T)*(1 - TLOSS) - TCOSTR(I,J)

- Again, an arbitrage condition

The same conditions hold for breakfast where we substitute

- BFAST.M for ROLLER.M

- TCOSTB for TCOSTR

- The same is true for the storage variables for the processed products
Maize Marketing System (cont’d.)

- The FOC for STORER(I,T) is

\[ \text{SCOSTR}(I) + \text{ROLLER}.M(I,T) \]
\[ - \text{ROLLER}.M(I,T+1) \cdot (1 - \text{SLOSS}) \]
\[ - \text{STORER}.M(I,T) > 0 \]
\[ \text{or,} \]
\[ \text{ROLLER}.M(I,T) \geq \text{SCOSTR}(I) \]
\[ + \text{ROLLER}.M(I,T+1) \cdot (1 - \text{SLOSS}) - \text{STORER}.M(I,T) \]

Maize Marketing System (cont’d.)

- and for STOREB(I,T) is

\[ \text{SCOSTB}(I) + \text{BFAST}.M(I,T) \]
\[ - \text{BFAST}.M(I,T+1) \cdot (1 - \text{SLOSS}) \]
\[ - \text{STOREB}.M(I,T) \geq 0 \]
\[ \text{or,} \]
\[ \text{BFAST}.M(I,T) \geq \text{SCOSTB}(I) \]
\[ + \text{BFAST}.M(I,T+1) \cdot (1 - \text{SLOSS}) - \text{STOREB}.M(I,T) \]
Maize Marketing System (cont’d.)

- There is one additional constraint that must be accounted for – processing capacity

- PROCLIM(I,T) ..
- \[- \frac{1}{YIELDR} \cdot MAKER(I,T) - \frac{1}{YIELDB} \cdot MAKEB(I,T) \]
- \[= G = - PCAP(I) \]

- This constraint is stated in tons of maize

Maize Marketing System (cont’d.)

- Now consider the price linkages due to processing

- The FOC for MAKER(I,T) is:

- MCOSTR(I) + MAIZE.M(I,T) \cdot 1/YIELDR
- \[- ROLLER.M(I,T) + PROCLIM.M(I,T) \cdot 1/YIELDR \]
- \[\geq 0 \]
Maize Marketing System (cont'd.)

- or,

\[
\text{ROLLER}.M(I,T) \leq \text{MAIZE}.M(I,T) \times \frac{1}{\text{YIELDR}} + \text{MCOSTR}(I) + \text{PROCLIM}.M(I,T) \times \frac{1}{\text{YIELDR}}
\]

- Again, we have an arbitrage condition

The analogous linkages for breakfast processing are:

The FOC for \( \text{MAKEB}(I,T) \) :

\[
\text{MCOSTB}(I) + \text{MAIZE}.M(I,T) \times \frac{1}{\text{YIELDB}} - \text{BFAST}.M(I,T) + \text{PROCLIM}.M(I,T) \times \frac{1}{\text{YIELDB}} \geq 0
\]

- or,

\[
\text{BFAST}.M(I,T) \leq \text{MAIZE}.M(I,T) \times \frac{1}{\text{YIELDB}} + \text{MCOSTB}(I) + \text{PROCLIM}.M(I,T) \times \frac{1}{\text{YIELDB}}
\]
Maize Marketing System (cont’d.)

- Note that all of these arbitrage conditions taken together result in a fully linked system of product flows and value:

  - Shipping variables move product across regions and limit price differentials between regions
  - Storage variables move product *forward* through time and limit upward price differentials across periods
  - Processing variables transform raw maize into one of the two products and limit price differentials between forms

Maize Marketing System (cont’d.)

- The forgoing model was complete except for

  - Initial stocks for each of the goods (maize, breakfast, and roller)
  - Outgoing stock requirements of the goods (if you have incoming stocks, it is typical to also have outgoing stock requirements)