Agricultural Economics 652

Read the entire exam before you begin. Put your name on every page. **Show all your work, and justify all your answers.** Minimal credit will be given for unjustified answers. Good luck!

(15)

1. We have spent some time in the course learning the GAMS language. One important part of debugging a program is figuring out what the program does and, in some cases, why it doesn’t succeed in doing what you have specified. Below you will find a bit of GAMS output that generates an error. Review the program before answering the following questions.

```
1 set
2   i       / apple,orange,dog,cat,cow,pig /
3 mammal(i) / cow,dog,cat,pig /
4   food(i) / pig,apple,orange,cow /
5   farm(i);
6 parameter product(i);
7   product(i) = ord(i);
8   product(food) = 3;
9   product(mammal) = 5;
10  farm(i)   = mammal(i)*food(i);
11  product(farm) = 7;
12  product(i) = product(i) - 5*mammal(i);
13  product(i) = 1/product(i);
```

**EXECUTION ERROR 0 AT LINE 13 .. DIVISION BY ZERO**

```
EXECUTION TIME       =        0.000 SECONDS    1.4 Mb      WIN-18-096
```

a. In the following table, indicate the contents of the parameter “product” after execution of the specified line number.

<table>
<thead>
<tr>
<th>Line #</th>
<th>Apple</th>
<th>Orange</th>
<th>Dog</th>
<th>Cat</th>
<th>Cow</th>
<th>Pig</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
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<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b. Why has an error been generated? Be as precise as possible in explaining exactly what causes the error.

(15)

2. We have also spent a fair amount of time talking about units in our formulations. Consider the following small GAMS program (read the comments too!).

```
3 option limrow=0,limcol=0;
4 set
5 animal Animals to be fed / cow,pig /
6 feedstuff Feed ingredients / corn pounds,soymeal pounds,
7                      chocolate pounds,hay tons /
8 nutrient Nutrients required / energy kilocalories,protein grams,
9                      lysine milligrams,
10                      drymatter kilograms /;
11 table req(nutrient,animal) Nutrient requirements per animal per day
                (units are specified in the declaration for nutrient - nutrient
                units per animal)
12                   cow    pig
13  energy     12      5
14  protein    30     20
15  lysine     20     18
16  drymatter .01        ;
17 table cont(feedstuff,nutrient) Nutrient content of feedstuffs (units
                are specified in the declarations for feedstuffs and nutrient - nutrient
                units per feedstuff unit)
18              energy  protein  lysine   drymatter
19  corn          1000        1       2        .005
20  soymeal       1500        2       5        .001
21  chocolate    20000       .1
22  hay                                         900 ;
23 parameter price(feedstuff) Price of feedstuff in dollars per pound
24             / corn 0.20,soymeal 0.45,chocolate 5,hay 0.01 /;
25 parameter head(animal) Number of animals of each type to be fed
26            / cow 3,pig 20 /;
27 variables
28   cost                Total cost in dollars to feed the animals ;
29 positive variables
30   x(feedstuff,animal) Total amount of each feedstuff to feed by type of
31   animal (units of feedstuff are in the feedstuff declaration) ;
32 equations
33   costdef               Total cost defining equation
34   nutreq(nutrient)       Nutrient requirements ;
35   costdef              .. cost =e= sum((feedstuff,animal),price(feedstuff)*x(feedstuff,animal)) ;
36   nutreq(nutrient)       .. sum((feedstuff,animal),cont(feedstuff,nutrient)*x(feedstuff,animal))
37 model feedmix / costdef,nutreq /;
38 solve feedmix using lp minimizing cost ;
```

There are several problems with the above formulation. Assume that the data is correct. Analyze the units for the problem and identify at least three inconsistencies. (Hint: the optimal solution to this problem as formulated is to feed 50 pounds of corn to the cow species for a total cost of $10. That cannot be right, can it?)

(20)

3. Consider a producer’s profit maximization problem (two outputs and two inputs) that has the following form:
maximize $\sum_{j=1}^{2} p_j y_j - c(y_1, y_2, w_1, w_2)$
subject to: $0 < l_i \leq y_i \leq u_i$, $i = 1, 2$

where $y_j$ denotes the outputs, $p_j (>0)$ denotes output prices, and $w_j (>0)$ denotes input prices. Further, let $c()$ have the following form:

$c(y_1, y_2, w_1, w_2) = 2w_1 y_1^2 + 3w_2 y_2^3$.

a. Is this a “nice” problem? Why? (Be sure to define nice. To get the most points, define nice the way we have defined it in class! Also, if you claim that the problem is nice by your definition, prove that it is nice.)

b. What about the solution to this problem is unique? (E.g., the optimal objective value? Optimal variable values? Optimal shadow prices?)

c. Think of the optimal solution to the above problem as a function of the prices of inputs and outputs. Is this function continuous? Is it differentiable? Why or why not?

(20)

4. When we discussed static problems with risk, we noted that this is one way to explain the fact that we observe people (decision-makers) diversifying their investment holdings. The simplest model of investment behavior is the mean-variance model. Consider the following mean-variance portfolio model:

$maximize 7x + 4y - \rho / [30x^2 + 5xy + y^2],$
subject to: $x + y \leq 100$

where $\rho$ is a parameter reflecting risk aversion. The optimal solutions for this problem for three different values of the risk aversion parameter are as follows:

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.000</th>
<th>0.002</th>
<th>0.020</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>100.000</td>
<td>51.923</td>
<td>0.000</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.000</td>
<td>48.077</td>
<td>100.000</td>
</tr>
</tbody>
</table>

a. Draw the mean-variance frontier for this problem (label your axes carefully, and indicate the coordinates of any special points on the graph).

b. Could the results in the table be correct? If so, explain why they make sense. If not, explain why they are not possible.
(20)

5. Consider the standard transportation problem:

\[
\text{minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]
subject to: \( \sum_{j=1}^{n} x_{ij} \geq d_j, \quad s_i \geq \sum_{j=1}^{n} x_{ij}, \quad x_{ij} \geq 0. \)

a. Write down the Lagrangian for this problem, and set up the first-order conditions with respect to \(x_{ij}.\)

b. Rearrange the first-order condition you derived in part a, and interpret it from an economic perspective.

c. Modify the formulation of the standard transportation model to reflect a 5% shipping loss from all sources \((i)\) to all destinations \((j)\). (Just indicate what changes and what remains the same from the standard formulation.

d. Derive the first-order conditions for \(x_{ij}\) for the modified problem and interpret it economically.

(10)

6. Consider the decision confronting a grain farmer of whether to build a hog barn on some of his land. An important aspect of this investment is that if he chooses to build the barn, he must spend a significant sum of money right away, and he can only remove the barn at significant expense and with little or no salvage value (i.e., he cannot really sell it). In a deterministic world, he could simply evaluate the present value of the net revenue stream for the hog barn versus the present value of the net revenue stream associated with keeping the land in crops, choosing the higher of the two. However in the real world, the prices of hogs and crops are dynamic and stochastic. Further, the farmer can choose to invest this year, next year, or in any other future year.

a. Keeping in mind that the life of a hog building is about 20 years, what methodology would you employ to evaluate the decision of whether to invest now or to wait? Why would you use the method you have chosen?

b. Explain what random variables you would consider explicitly and what decisions you would consider in your problem. (Please be explicit about what is dynamic and what is conditional in your response.)