Testing Random Effects in Two-Way Spatial Panel Data Models

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Abstract

This paper proposes an alternative testing procedure to the Hausman test statistic to help the applied researcher to determine the adequacy of the random effects specification in a two-way spatial autoregressive panel data model. Our model includes both individual and time specific effects to deal with unobserved heterogeneity. Moreover, we allow for interactions between cross-sectional units through the presence of an endogenous spatial lag (SAR). We suggest a two-step procedure that first partial out the spatial component of the model and then generalizes the original Mundlak approach to both individual and time effects. Three tests are proposed to find out the best way to model individual and time effects in the specification. This approach has the advantage of simplicity since all tests are based on FGLS estimations and standard Wald statistics can be applied. Some Monte Carlo simulations show the properties of this procedure in small sample.

Keywords: Spatial Autocorrelation, Panel Data, Random effects, Mundlak

JEL: C12; C21; C23; C52

1 Introduction

Spatial autoregressive (SAR) panel data models are of high interest since they allow capturing individual, temporal and interactive heterogeneity. The first two types of heterogeneity come from individual or time characteristics and are easily dealt with using either a fixed or random effects panel data framework. Interactive heterogeneity is due to differentiated feedback effects, originating from cross-section interactions between individuals. It cannot be dealt with traditional panel data methods and furthermore requires explicit modeling of spatial autocorrelation. More precisely, interactive heterogeneity is captured by impact coefficients or elasticities computed from the reduced form of the spatial panel data model taking into account the interaction structure between individuals.

Anselin (1988, chap.10) was the first to suggest a one-way random effects specification with spatially autocorrelated errors (Anselin’s model hereafter). Several likelihood ratio and Lagrange multiplier (LM) statistics were derived by Baltagi et al.

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1Interactive heterogeneity should not be confused with what literature labels spatial heterogeneity and which refers to standard individual heterogeneity coming from spatial structural instability in coefficients or residual variance.
(2003) to assess the relevance of random individual effects and spatial autocorrelation in this model. These statistics were further generalized to the presence of serial correlation by Baltagi et al. (2007). Also, Baltagi, Song & Kwon (2009) suggest LM statistics to test for the presence of heteroskedasticity and spatial autocorrelation in this Anselin’s model. Alternatively, Kapoor et al. (2007) (KKP hereafter) develop a method of moments estimator for a one-way random effects model where spatial autocorrelation is present in both components of the error term. These two different approaches lead Baltagi, Egger & Pfaffermayr (2009) to propose a generalized random effects model that encompasses KKP and Anselin’s model and to derive LM tests to choose between the two specifications. According to Lee & Yu (2009), this distinction is not important in the fixed effects specification since individual-specific effects enters the model as vectors of unknown fixed effects parameters. Recently, Lee & Yu (2010a,b) have established asymptotic properties of maximum likelihood estimation of a SARAR panel data model (i.e specification containing an endogenous spatial lag and spatially autocorrelated errors) with possibly both time and individual fixed effects.

Yet, the critical issue for the applied researcher is to determine the best specification for individual and temporal heterogeneity. To this aim, Hausman (1978) derived a statistic to evaluate the relevance of the random effects specification with respect to the fixed effects model. In the spatial panel data literature, only Mutl & Pfaffermayr (2010) and Lee & Yu (2009) study the use of such a Hausman type statistic but focus on the sole presence of individual heterogeneity. The former paper concentrates on the KKP specification while the interest of the latter lies in the Anselin model.

The originality of this paper is to suggest an alternative testing procedure to assess the relevance of a two-way random effects specification. Our procedure generalizes Mundlak’s (1978) approach to a SAR panel data model with both individual and time heterogeneity. Extending the Mundlak approach to SAR panel data model is however tricky since the dependence between individual and time effects and the endogenous spatial lag should be accounted for, leading to estimation difficulties. To avoid such complications, we propose a simple two-step strategy. In the first step, we partial out the SAR component by spatially filtering the spatial panel data model. The resulting non-spatial model boils down to a simple two-way panel model on which Mundlak’s extension to both individual and time effects can be applied. The Mundlak methodology consists in augmenting the two-way random effects non-spatial specification by variables that should capture the correlation between regressors and unobserved heterogeneity. We go one step further and test for the significance of these additional variables to assess the tradeoff between bias and efficiency.

The rest of the paper is organized as follows: Section 2 presents the model under study and the proposed tests. Monte Carlo simulations are conducted in Section 3 to assess the performance of the tests in finite sample and Section 4 concludes.

2 Model and tests statistics

Our benchmark model is a two-way error components SAR panel data model:

\[
\begin{align*}
Y_t &= \rho W Y_t + X_t \beta + U_t \\
U_t &= \mu + \phi u_t N + V_t \quad t = 1, \ldots, T
\end{align*}
\]
where $\mathbf{Y}_t = (y_{1,t}, y_{2,t}, \ldots, y_{N,t})$ is the $N$-dimensional vector of the dependent variable for all individuals in period $t$, $\mathbf{X}_t$ is the $N \times k$ matrix of exogenous variables, $\beta$ is the associated vector of unknown regression coefficients to be estimated. $\mathbf{V}_t = (V_{1,t}, V_{2,t}, \ldots, V_{N,t})$ is the innovation term, $V_{i,t}$ is i.i.d. across $i$ and $t$ with zero mean and variance $\sigma^2_{\nu}$, and $\mu$ is the $N \times 1$ vector of systematic effects associated with the $N$ “individuals”. $\phi_t$ is the time effect and $\iota_N$ is the unit vector of dimension $N$. $\mathbf{W}$ is a squared non-stochastic matrices of dimension $N$ typically referred to as interaction or spatial weight matrix and $\rho$ is the unknown spatial autoregressive parameter to be estimated.\footnote{Variables and weight matrix are assumed to form triangular arrays but for notations clarity, we will omit the $n$ index.}

According to Mundlak (1978), the random effects specification is a misspecified version of the fixed effects model since it ignores the possible correlation between individual effects and regressors. By controlling for this correlation, he shows that coefficients of the random effects specification are identical to those of the within estimation unifying in this way the two approaches. He thus proposes the use of an auxiliary regression that controls for this dependence and to insert those control variables to the original random effects specification.

By contrast to the original Mundlak approach, when we study SAR panel data models we cannot directly set the auxiliary regressions that will control for the possible correlation between individual and time heterogeneity and the regressors. The reason is that we cannot assume that individual and time effects are independent from the endogenous spatial lag $\mathbf{W}y$.\footnote{By definition, a time effect affects all individual units in a given period, for instance the recent financial crisis.} However, explicitly accounting for this potential correlation in the auxiliary regressions lead to problematic estimations. To avoid such difficulties, we propose a two-step procedure. The first stage consists in partiailling out the spatial component and get an a-spatial model. In the second step, we apply the Mundlak approach to both individual and time effects on this spatially filtered specification.

Lee & Yu (2010a,c) have shown that the two-way fixed effects spatial panel model provides consistent estimators for $\rho$ and $\beta$’s since the assumption of independence between effects and regressors is not set. We will thus use this $\rho$ estimator to filter out the spatial component of our model. Let us consider Lee & Yu (2010a) model without accounting for spatially autocorrelated errors:

$$\mathbf{Y}_t = \rho \mathbf{W} \mathbf{Y}_t + \mathbf{X}_t \beta + \mu + \phi_t \iota_N + \mathbf{V}_t \quad t = 1, \ldots, T \quad (2)$$

Depending on whether $T$ is finite or not, Lee & Yu (2010a) propose two different ways to estimate this model. In the first situation, $T$ finite, they get rid of individual effects only and consider time effects as a finite number of additional regression coefficients. The authors do not use the traditional deviation from mean operator $J_T = I_T - \frac{1}{T} \iota_T \iota_T'$, where $I_T$ is the identity matrix of dimension $T$ and $\iota_T$ a $T$-dimensional vector of ones, to remove individual effects but its orthonormal eigenvector matrix $[\mathbf{F}_{T,T-1}, \frac{1}{\sqrt{T}} \iota_T]$.\footnote{Lee & Yu (2010a) show that even though the within transformation developed in Elhorst (2003) provides consistent $\rho$ and $\beta$ coefficients, the variance of the disturbances is not consistently estimated when $T$ is small but $N$ large.} $\mathbf{F}_{T,T-1}$ is the $T \times T - 1$ matrix of eigenvectors of $\mathbf{J}_T$ corresponding to eigenvalues equal to unity while $\frac{1}{\sqrt{T}} \iota_T$ is the eigenvector associated to the null
eigenvalue. Their suggested transformation applies the \( F_{T,T-1} \) matrix to original data set:

\[
(Y_1', \ldots, Y_T')' = (F_{T,T-1} \otimes I_N)(Y_1', \ldots, Y_T').
\]  

We note that the transformed sample is shrunk of one period of time. Model (2) thus becomes:

\[
Y_t^* = \rho W Y_t^* + X_t^* \beta + \phi_t^* + V_t^* \quad t = 1, \ldots, T - 1,
\]  

where \([\phi_{1tN}, \phi_{2tN}, \ldots, \phi_{TtN}] = [\phi_{1tN}, \phi_{2tN}, \ldots, \phi_{TtN}]F_{T,T-1} \) are the transformed time effects.

For notational purposes, note that for any variable \( Z_t \), \( Z_t = Z_t - \frac{1}{T} \sum_{t=1}^{T} Z_t \). Even though the assumptions underlying the asymptotic properties of (quasi)-maximum likelihood estimators for model (2) are developed and discussed in Lee & Yu (2010c), we nevertheless state two of them because they change depending on the way to deal with time effects.

For this first case where time effects are estimated, these two assumptions, without spatially autocorrelated disturbances, are:

**Assumption 1.** \( W \) is a nonstochastic spatial weight matrix with zero diagonal.

**Assumption 2.** The elements of \( X_t \) are nonstochastic and bounded, uniformly in \( N \) and \( t \). Also, in the case where \( N \) is large and \( T \) is finite or large, the limit of \( \frac{1}{nT} \sum_{t=1}^{T} \hat{X}_t \hat{X}_t' \) exists and is nonsingular.

Defining \( S(\rho) = (I_n - \rho W_n), \theta' = (\beta', \rho, \sigma^2), \eta' = [\beta', \rho] \) and \( \phi_T' = [\phi_1^*, \ldots, \phi_{T-1}^*] \) we write the log-likelihood function of (4) as:

\[
lnL(\theta, \phi_T^*) = -\frac{n(T-1)}{2} \ln 2\pi - \frac{n(T-1)}{2} \ln \sigma^2 + (T-1) \ln |S(\rho)|
\]

\[
- \frac{1}{2\sigma^2} \sum_{t=1}^{T-1} V_t'(\eta, \phi_t^*)V_t'^* (\eta, \phi_t^*),
\]  

with \( V_t^*(\eta, \phi_t^*) = S_n(\rho)Y_t^* - X_t^* \beta - \phi_t^* \).

In their second approach, Lee & Yu (2010a) propose to further transform (4) to eliminate time effects. The traditional way to eliminate time effects is to take deviation from the cross-sectional mean, namely applying the \( J_N = I_N - \frac{1}{n} \sum_{t=1}^{n} \hat{X}_t \hat{X}_t' \) operator. However, the authors prefer a transformation based on the orthonormal eigenvector matrix of \( J_N \), namely \( (F_{N,N-1}, \frac{1}{\sqrt{n}} \hat{X}_t) \), where \( F_{N,N-1} \) is the \((N \times N-1)\) eigenvectors matrix corresponding to unit eigenvalues, because it does not generate linear dependence in the resulting disturbances. Applying this \( F_{N,N-1} \) to the \( N \)-dimensional vector \( Y_t^* \) of model (4) produces a \((N-1)\)-dimensional vector \( Y_t^{**} \) such that \( Y_t^{**} = F_{N,N-1} Y_t^* \). Let us note that the effective sample size after these 2 transformations is now \((N-1)(T-1)\). Finally, to estimate this transformed model by maximum likelihood, \( W \) needs to be row normalized, which leads to the modification of assumption 1 above.

**Assumption 1'.** \( W \) is a row normalized nonstochastic spatial weight matrix with zero diagonal.

Lee & Yu (2010a) show that applying the \( F_{N,N-1} \) operator on model (4) gives:

\[
Y_t^{**} = \rho (F_{N,N-1}’ W F_{N,N-1}) Y_t^{**} + X_t^{**} \beta + V_t^{**}, \quad t = 1, \ldots, T - 1.
\]
and the associated log-likelihood function is:

\[
\ln L(\theta) = -\frac{(N - 1)(T - 1)}{2} \ln 2\pi \sigma^2 - (T - 1) \ln (1 - \rho) \\
+ (T - 1) \ln |S(\rho)| - \frac{1}{2\sigma^2} \sum_{t=1}^{T-1} V^{**'}(\eta) \tilde{V}^{**}(\eta) 
\]

which Lee & Yu (2010a) have shown to be identical to:

\[
\ln L(\theta) = -\frac{(N - 1)(T - 1)}{2} \ln 2\pi \sigma^2 - (T - 1) \ln (1 - \rho) \\
+ (T - 1) \ln |S(\rho)| - \frac{1}{2\sigma^2} \sum_{t=1}^{T-1} \tilde{V}^{'}(\eta) J_N \tilde{V}(\eta) 
\]

where \( \tilde{V}_t = S(\rho) \tilde{Y}_t - \tilde{X}_t \beta \). This last expression provides motivation for the change in the second assumption:

**Assumption 2'** The elements of \( X_t \) are nonstochastic and bounded, uniformly in \( N \) and \( t \). Also, in the case where \( N \) is large and \( T \) finite or large, the limit of \( \frac{1}{nT} \sum_{t=1}^{T} \tilde{X}'_t J_N \tilde{X}_t \) exists and is nonsingular.

Specifications (4) and (6) provide consistent estimators for \( \rho \) and the \( \beta \)'s. We thus can use one of these two models (depending whether we prefer to estimate the time effects or not) to spatially filter model (1) and get the following expression:

\[
\hat{Y}_t = X_t\beta + U_t \\
U_t = \mu + \phi_t + V_t \quad t = 1, \ldots, T
\]

with \( \hat{Y}_t = Y_t - \hat{\rho} W Y_t \). As the spatial component has been partialled out, this model corresponds to a standard two-way panel data model that can be estimated by OLS.\(^5\)

Stacking the observations for all time periods yields:

\[
\tilde{Y} = X \beta + U \\
\tilde{U} = Z \mu + Z \phi + V
\]

where \( Z \mu = (\iota_T \otimes I_N) \) and \( Z \phi = (I_T \otimes \iota_N) \) are two matrices of dummy variables.

Using this spatially filtered model, we can now test the relevance of random effects specification using an extension of Mundlak (1978) while avoiding to model the correlation with the endogenous spatial lag. Let us introduce two auxiliary regressions that capture the possible dependence between unobserved heterogeneity and regressors:

\[
\mu_i = X_{i,t}\pi + \alpha_{i,t} \\
\phi_t = X_{i,t}\gamma + \omega_{i,t}
\]

Averaging over time gives:

\[
\mu_i = \bar{X}_{i,T} \pi + \bar{\alpha}_i \\
\phi_t = \bar{X}_{T,t} \gamma + \bar{\omega}_t
\]

\(^5\)Coefficients estimators using this spatial filtering approach were shown to be identical to a standard SAR estimation (Anselin 1988, pp. 181-182).
where $\bar{X}_i = 1/T \sum_{t=1}^{T} X_{i,t}$ is the average over time of $X_i$ for individual $i$; $\bar{X}_t = 1/N \sum_{i=1}^{N} X_{i,t}$ is the average over individuals of $X_i$ in period $t$; $\omega_t \sim N(0, \sigma^2_\omega)$ and $\alpha_i \sim N(0, \sigma^2_\alpha)$.

The independence assumption between individual effects and regressors is violated when $\pi$ is significant in equation (11). In such a case, model (10) should include individual fixed effects. However, when $\pi$ is not significant, individual heterogeneity in specification (10) is best modeled by random effects. The same argument applies for time effects (equation 12): when $\gamma$ is significant, specification (10) should include fixed time effects whereas if $\gamma$ is not significant, random time effects better model temporal heterogeneity. We can rewrite $Z_{\mu} \mu$ and $Z_{\phi} \phi$ as follows:

$$Z_{\mu} \mu = (\bar{J}_T \otimes I_N) (X \pi + \alpha)$$

$$= P_\mu (X \pi + \alpha)$$

$$Z_{\phi} \phi = (I_T \otimes \bar{J}_N) (X \gamma + \omega)$$

$$= P_\phi (X \gamma + \omega)$$

where $\bar{J}_T = I_T^T I_T / T$ is the operator computing averages of observations over time and $\bar{J}_N = I_N^T I_N / N$ is the operator computing averages of observations over individuals. Expressions (15) and (16) come from use of the projection matrix on the column space of $Z_{\mu}$ and $Z_{\phi}$, which are respectively $\bar{J}_T \otimes I_N$ and $I_T \otimes \bar{J}_N$. Plugging these last two equations into (10) yields:

$$\hat{Y} = X \beta + P_{\mu} X \pi + P_{\phi} X \gamma + U$$

$$U = P_{\mu} \alpha + P_{\phi} \omega + V$$

By construction, the error term of this last model is a two-way variance components model that has to be estimated by random effects.

The procedure we propose to assess the relevance of random effects specification is based on specification (17) and is made of three hypotheses.

i) $H^a_0 : \pi = \gamma = 0$

ii) $H^b_0 : \pi = 0$

iii) $H^c_0 : \gamma = 0$

The applied researcher should first test $H^a_0$, i.e the joint significance of $\pi$ and $\gamma$, in (17). When this hypothesis is not rejected, model (10) can be estimated with a two-way error components framework. However, if the hypothesis is rejected, one can conclude to the presence of correlation between unobserved heterogeneity and regressors but the test does not reveal whether it comes from characteristics peculiar to individuals, time or both. To answer this question, (ii) and (iii) have to be tested to identify the most appropriate specification. Three cases are of interest. First, if both hypotheses are rejected, model (10) should include fixed effects for both components. Second, if only $H^b_0$ is rejected, individuals fixed effects should be included in model (10) while temporal heterogeneity is best modeled by an error component type. Last, if only $H^c_0$ is rejected, temporal heterogeneity should be modeled by fixed effects while an error component better models individual characteristics. These last two cases are known in the literature as mixed-effects models (see, among others Abowd et al. 1999, Andrews et al. 2006). These 3 tests can be viewed as an arbitrage between bias and efficiency. For instance, if the adopted model is the random effects
specification, one prefers a model with a small (negligible) bias but efficient, meaning that we favor a slightly biased estimator but much more accurately estimated. Let us finally note that since model (17) is estimated by (feasible) GLS, tests for the three hypotheses are standard Wald statistics. Finally, one just has to estimate again the SAR panel data once the best way to model the effects has been determined.

3 Monte Carlo simulations

To assess the finite sample properties of the proposed testing procedure, some Monte Carlo experiments are conducted. We set the data generating process as follows:

\[ y_{i,t} = \rho W_{i,t} y_t + \beta x_{i,t} + u_{i,t} \]
\[ u_{i,t} = \mu_i + \phi_t + \nu_{i,t}, \]

where \( W_{i,t} \) is the \( i \)th row of the spatial weight matrix associated to the endogenous spatial lag while \( \rho \) vary over the set \([-0.6, 0.6]\) by increment of 0.2 and \( \beta \) is set to 1. \( x_{i,t} = \alpha_i + \lambda_t + \nu_{i,t} \), meaning that the scalar explanatory variable is composed of a time constant, an individual constant and an individual-time varying component, all drawn from independent standard normal distributions. Also, \( \nu_{i,t} \sim \mathcal{N}(0, 1) \).

To test the 3 hypotheses above, we consider 2 panel sizes based on regular grid \((N = 36, 49)\) while \( T \) is equal to either 20, 30 or 60. The spatial weight matrix considered is a row normalized rook contiguity of order 1. The reason is that for these simulations, the estimation of \( \rho \) is based on the second approach developed by Lee & Yu (2010a), namely the elimination of both individual and time fixed effects. Experiments are replicated 1000 times and performed in Matlab.

Four different DGP were considered for the error component processes which are summarized in Table 1. The first considers correlation of both effects with the regressor, the second, random time effects and correlated individual effects, the third is the other way around while DGP 4 assume full random specification. For each of these 4 DGP’s, the three hypotheses were systematically tested and Tables 2 to 4 present the rejection rate (power) of the associated tests for all the cases considered. Let us note that under the null, the rejection rate of a test corresponds to its size, which, for \( H_0^b \) and \( H_0^c \), is set to 5\%.

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<tr>
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<th>DGP2</th>
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In the simulations, \( \eta_i \sim \mathcal{N}(0, 2) \) and \( \epsilon_t \sim \mathcal{N}(0, 3) \). Also, \( \pi \) and \( \gamma \) are set to 1.

regressor, the second, random time effects and correlated individual effects, the third is the other way around while DGP 4 assume full random specification. For each of these 4 DGP’s, the three hypotheses were systematically tested and Tables 2 to 4 present the rejection rate (power) of the associated tests for all the cases considered. Let us note that under the null, the rejection rate of a test corresponds to its size, which, for \( H_0^b \) and \( H_0^c \), is set to 5\%. Table 2 presents results for \( T = 20 \) and for the 2 panel sizes selected. Let us look at the \( N = 36 \) case. When both errors components are correlated with the regressor (DGP 1), we observe a good power of the 3 tests, \( W a^a \), \( W a^b \) and \( W a^c \) even in a small sample size. We also note that the power is not affected by the value of the spatial autoregressive parameter. For DGP 2, where correlation is assumed only between individual effects and the regressor, \( W a^c \) is rightly sized. Also, power of \( W a^b \) is close to one, meaning nearly full rejection of the null while the power of the joint test, \( W a^a \) is slightly lower. Results concerning DGP 3 indicate a right size for \( W a^b \). We nevertheless notice
lower power for $W_a^a$ and $W_a^c$. This can be due to the small $T$ size, which prevents to build true correlation with the regressor. This fact is even worse when the panel size increases to 49 since the relative time span is even lower. In DGP 4, where true random effects components were generated, sizes of the three statistics match their theoretical counterpart.

Table 3 presents the results for $T = 30$. when $N = 36$, we observe similar results to Table 2. Very good power for the three tests in DGP 1, a rightly sized $W_a^c$ test in DGP 2 while power of the tho other tests is also high, even though the rejection rate of $W_a^a$ is smaller. For DGP 3, the size of $W_a^b$ matches the theoretical 5%. We also observe a poor power for the joint statistic, even though the rejection rate increases with respect to Table 2, indicating that the size of the time span matters. The power of $W_a^c$ also raised with respect to Table 2. GDP 4 provides the same conclusions as in previous Table, namely adequate sizes for all the three statistics.

Analysis of Table 4 reveals that when the time span increases, ($T = 60$), tests behave better. For instance, for DGP 3, we notice much larger power for $W_a^a$ and $W_a^c$ compared to the 2 preceding Tables. Also, in DGP 1, a much higher power is observed for $W_a^c$ than before.

Let us finally mention that even not shown in the Monte Carlo results, the $\beta$ coefficient for the augmented spatially filtered random effects specification (17) and for the spatially filtered fixed effects model are numerically equivalent.

4 Conclusion

This paper proposes a strategy for the applied researcher to choose between a full fixed, full random or mixed effects spatial autoregressive model. In such, it constitutes an alternative to the well-known Hausman statistic. However, our approach has the advantage of simplicity and does not require the efficiency of the random effects specification. Our procedure consists in first partialling out the spatial component of the model by applying a spatial filter. To do so, we use a consistent $\rho$ estimator, provided by the full fixed effects specification (see Lee & Yu 2010a, for details). Second, we apply a generalization of the Mundlak methodology to this non-spatial model that will orient the researcher on the best way to model individual and time heterogeneity. Monte Carlo experiments show that the tests proposed have good power in finite sample but perform better with larger samples.

References


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<table>
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<th>ρ</th>
<th>W_α^a</th>
<th>W_α^b</th>
<th>W_α^c</th>
<th>W_α^a</th>
<th>W_α^b</th>
<th>W_α^c</th>
<th>W_α^a</th>
<th>W_α^b</th>
<th>W_α^c</th>
<th>W_α^a</th>
<th>W_α^b</th>
<th>W_α^c</th>
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<td>0.403</td>
<td>0.049</td>
<td>0.502</td>
<td>0.055</td>
<td>0.04</td>
<td>0.045</td>
</tr>
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<td>0.981</td>
<td>0.992</td>
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<td>0.398</td>
<td>0.047</td>
<td>0.491</td>
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<td>0.06</td>
<td>0.057</td>
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<td>0.994</td>
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</tr>
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<td>0.992</td>
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<td>0.064</td>
<td>0.489</td>
<td>0.066</td>
<td>0.055</td>
<td>0.074</td>
</tr>
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<td>0.999</td>
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<td>0.993</td>
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<td>0.074</td>
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<td>0.075</td>
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W_α^a tests \( \pi = 0 \), W_α^b tests \( \pi = 0 \) and W_α^c tests \( \gamma = 0 \).
Table 3: Tests for the relevance of the random effects specification, T=30

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<th>DGP 3</th>
<th>DGP 4</th>
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<tbody>
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<td></td>
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<td>Wa^a</td>
<td>Wa^b</td>
<td>Wa^c</td>
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<table>
<thead>
<tr>
<th></th>
<th>DGP 1</th>
<th>DGP 2</th>
<th>DGP 3</th>
<th>DGP 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Wa^a</td>
<td>Wa^b</td>
<td>Wa^c</td>
</tr>
<tr>
<td>N=49, T=30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1</td>
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<td>0.996</td>
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</table>

W_a^a tests π = γ = 0, W_a^b tests π = 0 and W_a^c tests γ = 0.
Table 4: Tests for the relevance of the random effects specification, T=60

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<th>$W_\alpha^b$</th>
<th>$W_\alpha^c$</th>
<th>$W_\alpha^a$</th>
<th>$W_\alpha^b$</th>
<th>$W_\alpha^c$</th>
<th>$W_\alpha^a$</th>
<th>$W_\alpha^b$</th>
<th>$W_\alpha^c$</th>
<th>$W_\alpha^a$</th>
<th>$W_\alpha^b$</th>
<th>$W_\alpha^c$</th>
</tr>
</thead>
<tbody>
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<td>0.984</td>
<td>0.992</td>
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<td>0.996</td>
<td>0.057</td>
<td>0.997</td>
<td>0.073</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
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<td>0.989</td>
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<td>0.983</td>
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<td>0.997</td>
<td>0.046</td>
<td>0.045</td>
<td>0.04</td>
</tr>
<tr>
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<td>0.988</td>
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<td>0.976</td>
<td>0.044</td>
<td>0.993</td>
<td>0.067</td>
<td>0.07</td>
<td>0.043</td>
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<tr>
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<td>0.977</td>
<td>0.976</td>
<td>0.993</td>
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<td>0.986</td>
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<td>0.04</td>
</tr>
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<td>0.975</td>
<td>0.066</td>
<td>0.072</td>
<td>0.056</td>
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<td>0.98</td>
<td>0.96</td>
<td>0.98</td>
<td>0.047</td>
<td>0.976</td>
<td>0.067</td>
<td>0.986</td>
<td>0.072</td>
<td>0.044</td>
<td>0.06</td>
</tr>
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<td>0.974</td>
<td>0.992</td>
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<td>0.99</td>
<td>0.063</td>
<td>0.063</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$W_\alpha^a$ tests $\pi = \gamma = 0$, $W_\alpha^b$ tests $\pi = 0$ and $W_\alpha^c$ tests $\gamma = 0$. 