MICROECONOMICS

July 2006

Part I

Instructions

Answer all parts of all questions. Be sure to allocate your time appropriately.

Show your work clearly and explain your answers briefly but in enough detail that your reasoning can be followed. No credit will be given for answers without appropriate reasoning. Do only the work requested. Be sure your answer is organized in a way that can be followed by the graders.

If you become stuck on a part of a problem or make an algebra error that prevents you from solving a part of a problem, explain how you would complete the rest of the problem. Be very specific.

To avoid loss of clarity when your answers are photocopied for the graders, be sure to use dark pencil or pen, write clearly, and do not write near the edge of the paper.
1. An individual has preferences represented by the twice continuously differentiable utility function $u(x, L)$, where $x$ is the amount of a single physical commodity and $L$ is hours of leisure. Marginal utilities are strictly positive everywhere: $u_x > 0$ and $u_L > 0$ for all non-negative $x$ and $L$. The individual has an endowment consisting of $T$ hours and no physical commodity. She is able to work at a job that pays $w$ per hour and to purchase the physical commodity for $p$ per unit. Consumption of the physical commodity takes time: it takes $tx$ hours of time to consume $x$ units of the physical commodity, where $t > 0$ is fixed.

a. If the individual consumes $x$ units of the physical commodity and $L$ hours of leisure, how many hours does she have available to work? What is the budget constraint faced by the individual?

b. Write and interpret the first-order conditions for an interior solution ($x > 0$ and $L > 0$).

c. Write the second-order condition for an interior solution to this problem.

d. At an interior solution to this problem, must the utility function satisfy the conditions on the Bordered Hessian for quasi-concavity [at the optimal point, not in general]?

e. Suppose the time endowment, $T$, changes (for example, by the introduction of some required, time-consuming tasks that do not affect the person’s preferences). Starting from an interior solution, how does the chosen amount of leisure time change? [You do not need to sign the answer, but you must provide a specific expression using the terms from this problem.]
2. A risk-neutral investor with initial wealth $20 million is considering the purchase of a small company for $10 million. Before making the purchase decision, if desired, the investor could hire an auditor to check the company’s financial records. If the records are truthful, the investor believes the company would be worth $11 million, but if they are fraudulent, the investor believes the company would be worth only $0.5 million. The investor believes there is a 20% chance the records are fraudulent.

If the investor hires the auditor, the audit of the financial records will definitely provide a "clean" report if there is no fraud. If the books are fraudulent, the audit will catch and report the fraud with probability \( p \), but with probability \( 1 - p \) the audit will miss the fraud and mistakenly provide a "clean" report.

a. How good must the auditor be (that is, how large must \( p \) be) in order for the investor to be willing to pay $100,000 to hire the auditor?

b. Suppose the investor were risk averse, with von Neumann-Morgenstern utility function \( u(x) = \ln(x) \), where \( \ln(\cdot) \) is the natural logarithm function. How much of the analysis from part (a) can be used here? Explain. Also explain what needs to be changed. Be sure to include an explicit equation (using the data from this problem) that defines the lowest possible \( p \). [You do not need to solve for the value of \( p \). Just provide the defining condition.]

c. Return to the risk-neutral investor from part (a). Assume the auditor always catches fraud \( (p = 1) \). How high a fee could the auditor charge and still be hired?

d. With the risk-averse investor from part (b), how would your answer to part (c) change? [As in part (b), providing an explicit defining equation using the data from this problem is enough.]
3. Consider two related competitive markets, for goods $x$ and $y$. The consumer sector is made up of a continuum of infinitesimal consumers with mass one. Each consumer will either purchase one (infinitesimal) unit of $x$ and no $y$, or purchase one (infinitesimal) unit of $y$ and no $x$, or purchase nothing, depending on the prices. A consumer of type $t$ has valuations

$$v(x,t) = t^2 - 6t^2 + 11t$$
for one unit of $x$ [useful fact: $v(x,t) = (t - 1)(t - 2)(t - 3) + 6$] and

$$v(y,t) = 4 - t$$
for one unit of $y$,

where the valuations act as reservation values for a single unit, with the understanding that a consumer never consumes both $x$ and $y$ (i.e., if a consumer were to consume more that one unit of a good or to consume both goods, then the total value obtained would be equal to the valuation of the single “best” unit consumed).

Suppose consumer types are distributed uniformly over the interval $[0, 4]$ [note the density of types is 0.25 over the interval] and in the initial allocation of goods, each $t$ with $3 \leq t \leq 4$ owns a unit of $y$ [so mass 0.25 of $y$] and each $t$ with $0 \leq t \leq 2$ owns a unit of $x$ [so mass 0.5 of $x$].

a. If the consumers are allowed to trade, what are the competitive equilibrium prices, which $t$ end up consuming one unit of $x$, and which $t$ end up consuming one unit of $y$? Explain your approach to solving the problem [not just your final answer]. Your explanation should include ideas for approaching a general problem as well as specific details for this problem.

b. Explain how the competitive equilibrium prices and allocation of goods would change if the mass of $y$ were slightly larger (for example, if in the initial allocation, each $t$ with $3 - \varepsilon \leq t \leq 4$ owns a unit of $y$, where $0 < \varepsilon < 1$). You do not need numerical answers. Just provide a detailed, specific description of the changes and the reasoning behind them.