

An Implicitly Additive Demand System

- The AIDADS specification of utility is an example of an implicit relationship
- It is typically stated in a primal form as an implicit function of utility and consumption quantities

$$G(U, x_1, x_2, \dots, x_n) = \sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(U)}{1 + \exp(U)} \ln(x_i - \gamma_i) - U = k$$

AIDADS Utility (Cont'd.)

- Notice that this is an *implicit* relationship between utility, demand and income.
- To ensure global good behavior

$$0 \leq \alpha^i, \beta^i \leq 1, \sum_{i=1}^n \alpha^i \leq 1, \sum_{i=1}^n \beta^i \leq 1, \text{ and } \gamma^i \geq 0.$$

- Global good behavior means:
 - | Demands vary as continuous functions of prices.
 - | Demands are strictly greater than subsistence.

AIDADS Utility (Cont'd.)

- AIDADS is best viewed as a generalization of the LES which is in turn a generalization of the Cobb-Douglas

Let $\alpha_i = \beta_i$, and $\gamma_i = 0$:

$$\sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(U)}{1 + \exp(U)} \ln(x_i - \gamma_i) - U$$
$$= \sum_{i=1}^n \alpha_i \ln(x_i) - U = k$$

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AIDADS Utility (Cont'd.)

- Rearranging this and making a strictly increasing transformation on utility yields:

$$v(x^1, \dots, x^n) = \prod_{i=1}^n x_i^{\alpha_i}$$

- which you should recognize as the Cobb-Douglas
- What about LES?

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AIDADS Utility (cont'd.)

- To get the LES we do not assume zero for the subsistence levels:

Let $\alpha_i = \beta_i$:

$$\sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(U)}{1 + \exp(U)} \ln(x_i - \gamma_i) - U$$
$$= \sum_{i=1}^n \alpha_i \ln(x_i - \gamma_i) - U = k$$

AIDADS Utility (cont'd.)

- Rearranging this and making a non-decreasing transformation on utility yields:

$$v(x^1, \dots, x^n) = \prod_{i=1}^n (x_i - \gamma_i)^{\alpha_i}$$

- which you should recognize as the LES

AIDADS Utility (cont'd.)

- AIDADS is more general than Cobb-Douglas or LES

- Income above subsistence is called discretionary income

$$I_D = I - \sum_{i=1}^n p_i \gamma_i$$

- Discretionary budget shares are constant for fixed utility (income if optimal)

$$s_i = \frac{\alpha_i + \beta_i \exp(U)}{1 + \exp(U)}$$

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AIDADS Utility (cont'd.)

- The utility maximization problem is stated a bit differently to account for the implicit relationship:

maximize U
 $x_i > \gamma_i, U$

subject to :

$$\sum_{i=1}^n \frac{\alpha_i + \beta_i \exp(U)}{1 + \exp(U)} \ln(x_i - \gamma_i) - U = k$$

$$\nabla p_i x_i < I.$$

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AIDADS Utility (cont'd.)

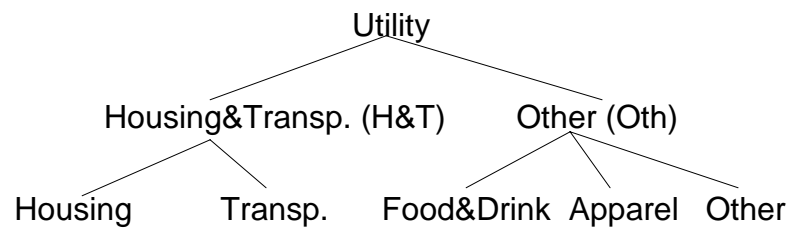
- As with Cobb-Douglas and the LES,
 - U is a concave (implicit) function of input levels x_i
 - Optimal levels of demand are strictly greater than the subsistence levels
 - Optimal levels of demand change as continuous, differentiable functions of prices
- As with the LES, discretionary income must be positive

Benchmarking the AIDADS?

- Consider benchmarking for AIDADS
 - How many parameters are there? $(3n+1)$
 - How many relationships do we get from a "snapshot" data set? (n)
 - There are $2n+1$ parameters left over
 - Research into benchmarking methods are active

Nested Utility Functions

- Nesting builds up the utility function by grouping goods together to form aggregate goods
- The nesting structure is often described by a tree. For instance for our 1994 consumption data:



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Nested Utility Functions (Cont'd.)

- Of course, this is just one possible nesting structure
- The top node of the tree is called “the root”, the lines are called “branches,” and the terminal nodes at the bottom of the tree are called “leaves”
- The part of the tree below a given node is called a “sub-tree”
- Properties of nested utility functions depend on the properties of the individual nesting functions

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Benchmarking Nested Utility Functions

- Benchmarking nested utility functions proceeds from the leaves towards the root
 - l The “sub-utility functions” associated with each sub-tree whose members are only leaves are benchmarked
 - l Then, prices and quantities are constructed for the sub-utility aggregates

Benchmarking Nested Utility (Cont'd.)

- l Once all sub-utilities have been benchmarked and prices/quantities for the aggregates have been constructed, the next higher level of sub-utility is benchmarked.
- l This process proceeds until the root has been reached.
- l Let us examine this process for our 1994 consumer data using the tree above.

Benchmarking Nested Utility (Cont'd.)

- I Further, let us assume the following elasticities of substitution
 - I Housing (H) and Transportation (T) is 1.2
 - I Food&Drink (F), Apparel (A), and Other (O) is 0.8,
 - I Housing and Transportation (HT) and Other (FAO) aggregates is 0.9.
- (Remember the formula $\rho = (1-\sigma)/\sigma$.)

Benchmarking Nested Utility (Cont'd.)

- I Thus, the relevant function to benchmark is:

$$(x) = \beta^{HT} \left[\beta^{HX^H} + \beta^{TX^T} \right]$$

$\begin{matrix} 0.167 & & 0.167 & -6/9 \\ -0.25 & & -0.25 & \\ & & & -0.25 \end{matrix}$

- I We will begin by benchmarking the two lower-level nests for Housing and Transportation, and Food&Drink, Apparel and Other.

Benchmarking Nested Utility (Cont'd.)

- Proceeding exactly as in the un-nested case, we find:

$$\beta_H = 0.605 \text{ and } \beta_T = 0.395$$

and

$$\beta_F = 0.277, \beta_A = 0.075 \text{ and } \beta_O = 0.649.$$

- The next step is to generate quantities and prices for the aggregates to allow us to benchmark the next level (as though it were a single level utility function).

Benchmarking Nested Utility (Cont'd.)

- We get the “pseudo-quantities” for the aggregates by simply applying the sub-utility function to the benchmark quantities. That is,

$$x_{HT} = (0.605 \times 10106^{1/6} + 0.395 \times 6044^{1/6})^6 = 8294.168$$

and

$$x_{FAO} = (0.277 \times 4689^{-1/4} + 0.075 \times 1644^{-1/4})^{-4}$$

Benchmarking Nested Utility (Cont'd.)

- Prices for the aggregates are then obtained by dividing the value of all goods in the aggregate by the pseudo-quantity computed above.

$$p_{HT} = (1 \times 10106 + 1 \times 6044) / 8294.168 = 1.947$$

and

$$p_{FAO} = (1 \times 4689 + 1 \times 1644 + 1 \times 9268) / 6523.007 = 2.392$$

- Q: Why am I carrying so many digits in the benchmarking process?

Benchmarking Nested Utility (Cont'd.)

- Now we have a full set of prices and quantities for the top-level utility benchmarking problem. Applying the usual formulas the the prices/quantities of the aggregates yields:

Benchmark Verification

- How do you know if you got it right?
 - If we have done the benchmarking correctly, then we should be able to set up the utility maximization problem with benchmarked parameters, benchmark income level, and benchmark prices; solve it, and get back the benchmark quantities.
 - (Does anyone really do this? How accurate should it be?)

Modifications of Consumer Problems

- Constraints -- Consumer's may not have full flexibility in their choices. Here are some examples of constraints that impact consumers.
 - Subsistence constraints
 - Prohibitions/rationing
 - Restrictions on expenditures for certain classes of goods (food stamps)

Mods. of Consumer Problems (Cont'd.)

■ Subsistence constraints

- These constraints are used to reflect the fact that consumers do not have full flexibility in choosing their consumption bundles. Even if food is expensive, consumers must have enough food to subsist.
- There are two approaches to subsistence constraints:
 - | Bounds on the consumption of (groups of) goods
 - | Modification of the utility function

Mods. of Consumer Problems (Cont'd.)

■ Bounds on consumption of (groups of) goods

- | This is the most commonly applied approach.
 - Typically, lower bounds are applied to the consumption of staples such as individual grains.
 - A minor modification is to apply the lower bound to the a weighted sum of consumption of several goods in order to reflect minimum requirements for basic nutrients such as calories or protein. (How would the units work here?)

Mods. of Consumer Problems (Cont'd.)

- | Implementation of the bound approach is simple -- merely impose lower bounds on consumption of individual goods

- | Or, if the lower bound is on consumption of some nutrient, impose general constraints that require adequate consumption of the nutrient.

Mods. of Consumer Problems (Cont'd.)

- Modification of the utility function to reflect subsistence requirements
 - | This approach is based on transforming the utility function so that the marginal utility of the good (nutrient) approaches infinity as the consumption of the good approaches the subsistence level.

 - | This is implemented by choosing a utility function that always produces strictly positive demands for goods and translating consumption by the subsistence quantity.

Mods. of Consumer Problems (Cont'd.)

- I Consider an example.
 - I Recall that the consumer with Cobb-Douglas utility always desires a positive quantity of every good.
 - I If we modify the Cobb-Douglas utility function by translating its argument by the subsistence quantity, then the consumer maximizing the modified utility function will always demand an amount of every good that exceeds the subsistence level.

Mods. of Consumer Problems (Cont'd.)

- I The modified utility function looks like:

$$(x^1, x^2, \dots, x^n) = \prod_{i=1}^n (x^i - \gamma^i)^{\beta_i}$$

- I Note that this is identical to the Cobb-Douglas except for the translation of the consumption quantities by γ_j .
- I Why will the optimal values for x_i exceed γ_i ?

Mods. of Consumer Problems (Cont'd.)

- | Note that our example corresponds to the well-known Linear Expenditure System (LES or Stone-Geary).
- | The γ_i are referred to as the subsistence quantities.
- | The β_i are need to be reinterpreted as the “discretionary budget shares” -- the shares of the budget beyond what is required for subsistence that is spent on good i .

Mods. of Consumer Problems (Cont'd.)

- | How would you benchmark an LES utility function (assuming you know the subsistence quantities)?
- | What would the analogous CES case look like?

Mods. of Consumer Problems (Cont'd.)

■ Prohibitions/rationing

- These are pretty simple -- they correspond to upper bounds on (groups of) consumption goods.
- The difference is that
 - | A prohibition suggests an upper bound of zero
 - | Rationing suggests a positive upper bound

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Mods. of Consumer Problems (Cont'd.)

- Historically in this country, we have had
 - | Prohibition on the consumption of alcohol (during the period known as "Prohibition")
 - | Rationing of gasoline during war times
- History question: Given what you know about the above two periods, is the suggested implementation an accurate model of what occurred? Why?
- How do we improve our model?

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Mods. of Consumer Problems (Cont'd.)

- Restrictions of expenditures on certain classes of goods
 - At times governments will try to influence spending patterns by making transfer payments that are not perfectly flexible. For example,
 - | The U.S. food stamp program
 - | The Japanese government distributed coupons for consumer expenditures with expiration dates

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Mods. of Consumer Problems (Cont'd.)

- Effectively, these transfer payments augment the budget constraint, but
 - | Since they cannot be spend without restriction, they create new constraints in the consumer's problem
 - | The restriction is an additional budget constraint limiting expenditures on non-target goods
 - (If I have \$100 cash and \$50 food stamps, how much can I spend on food? On other goods?)

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Mods. of Consumer Problems (Cont'd.)

■ Modifications of the budget constraint

■ How would you reflect

- | Good-specific taxes? (E.g., alcohol and tobacco)
- | Income taxes?
- | Lump sum transfers to consumers?
- | Subsidies? (E.g., energy investment tax credits)

Mods. of Consumer Problems (Cont'd.)

■ Other Difficulties

■ How does the perspective presented work for

- | Consumption of automobiles?
- | Either/or choices regarding taxes? (E.g., to itemize, or not to itemize.)

■ How should we decide when a decision is lumpy?