Model Building

- Optimization models — a means for estimating the impact of changes on an economic system that lie outside the range of historical data
  - Econometrics is often used for this as well — forecasting
  - When regime's change due to changes in policy or technology, econometric tools may be difficult to adapt to the task

Model Building (Cont'd.)

- Optimization models can proceed in this evaluation by the use of
  - "Reasonable approximations" to agent behavior
  - Engineering process data (for technology)
  - Representations of policy and environmental changes

Single-Agent, Static, Deterministic Models

- Consumers -- Neoclassical theory views consumers as maximizers of utility subject to (at least) a budget constraint.

$$\underset{x_1, x_2, \ldots, x_n}{\text{maximize}}\, U(x_1, x_2, \ldots, x_n)$$

$$\text{subject to}: \sum_{i=1}^{n} x_i p_i \le I,\ x_i \ge 0.$$

Single-Agent Models (Cont'd.)

where x_i denotes consumption of good i, p_i denotes the price of good i, and I denotes income.

The utility function, $U()$, is typically selected to be monotonically strictly increasing in each of its arguments and concave.

Single-Agent Models (Cont'd.)

- The above is a primal version of the consumer's problem – there are also dual formulations
  - For instance, one can base the problem on the "expenditure function," which is a function of price levels and the level of utility
  - Optimization of consumption levels is implicit with this function, and the consumer's problem becomes:

Single-Agent Models (Cont'd.)

$$\text{maximize } U$$
$$\text{subject to}: E(p_1, p_2, \ldots, p_n, U) \leq I.$$

- Note that the choice of the consumption bundle has now become implicit
- Levels of consumption are reclaimed by differentiation of the expenditure function and an envelope theorem

Single-Agent Models (Cont'd.)

- The other common dual formulation of the consumer's problem is based on the "indirect utility function,"

$$(p_1, p_2, \ldots, p_n, I)$$

- which yields the maximum level of utility as a function of the price levels and income. Note that none of the arguments of *V*() are choice variables for the consumer!

Single-Agent Models (Cont'd.)

- Consumer models are motivators for econometric demand systems
- Consumer models are also used directly as part of a "household model" wherein production and consumption decisions cannot be separated due to:
  - Incomplete markets for factors of production or consumption goods, and/or
  - Non-constant prices for consumption goods in models with risk

Benchmarking

- *Benchmarking* the primal consumer's problem
  - Econometric estimation is one approach to parameterizing consumer utility functions.
  - Another approach -- benchmarking.

    The idea of benchmarking is to choose values for the utility function parameters so that the optimality conditions for the consumer's utility maximization problem are exactly satisfied for given values for prices and consumption levels.

Benchmarking (Cont'd.)

- Mathematically, we divide the parameters in the utility function into two categories:
  - Ones that are set through literature survey/estimation (α_j, j =1,…,m) and
  - Ones that are set via benchmarking (β_i, i=1,…,n).
  - It is important that the number of parameters set by benchmarking exactly matches the number of consumption goods.

Benchmarking (Cont'd.)

- Consider the system of first-order conditions:

$$\frac{\partial U(x_1,\ldots,x_n)}{\partial x_i} - \lambda p_i = 0 \qquad i = 1,2,\ldots,n$$

$$\sum_{i=1}^{n} p_i x_i = I.$$

  - This is a system of *n*+1 equations. What are the variables?

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Benchmarking (Cont'd.)

- What happens if we
  - substitute known (benchmark) values for prices (p_i) and
  - quantities (x_i) into the first *n* equations above (along with the values of alpha from the literature), and then
  - solve for the betas?

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Cobb-Douglas Utility

- The utility function may be stated as:

$$(x_1,\ldots,x_n) = \alpha \prod_{i=1}^{n} x_i^{\beta_i} = \alpha x_1^{\beta_1} x_2^{\beta_2} \ldots x_n^{\beta_n}$$

- Without loss of generality, one can assume that α=1 and that the exponents sum to 1. (Why?)

Cobb-Douglas Utility (Cont'd.)

- Consumer behavior with Cobb-Douglas utility:
  - Strictly positive prices result in consumer demands
    - that are strictly positive for each good,
    - that exhaust income, and
    - that are continuously differentiable with respect to changes in relative prices.
  - What happens if prices are not strictly positive?

Benchmarking a Cobb-Douglas

- Recall that the goal is to choose parameter values so that the FOC's are satisfied for given prices and quantities
  - Note there are exactly enough parameters to satisfy the FOC's.
  - The solution to the FOC system when the β_i's are treated as the variables is:

$$\beta_i = \frac{p_i x_i}{I} = p_i x_i / \sum_{j=1}^{n} p_j x_j$$

Benchmarking Cobb-Douglas -- Example

- 1994 Consumer Expenditures

	Value	Price	Quantity
Food&Drink	4,689	1.00	4,689
Apparel	1,644	1.00	1,644
Housing	10,106	1.00	10,106
Transport.	6,044	1.00	6,044
Other	9,268	1.00	9,268
Total	31,751		

Source: Statistical Abstract of the United States, 1996, Table 704.

Benchmarking Example (Cont'd.)

- All the prices have been set to 1.0. Why?
- Using our benchmarking rules the parameters are:

$$\alpha = 1,\ \beta_{Food\&Drink} = 0.148,\ \beta_{Apparel} = 0.052$$
$$\beta_{Housing} = 0.318,\ \beta_{Transport.} = 0.190,\ \beta_{Other} = 0.292$$

Implementation Issues -- Powers

- We should be nervous whenever we see functions that are not defined for all values of its arguments like y^x.
  - What do these mean?
    8^3 = ? $4^{1/2}$ = ? 2^{-1} = ? 0^{-1} = ? $(-1)^{1/3}$ = ?
  - For what values of *y* and *x* does y^x give us trouble?

Implementation Issues -- Powers (Cont'd.)

- How does the computer evaluate these?
  - If x is not a whole number $y^x = \exp(x \ln(y))$.
  - If x is a whole number $y^x = y\ y \ldots y$ (with x terms) if x is positive and $y^x = 1/(y\ y \ldots y)$ (with x terms) if x is negative.
- What does this mean for implementation of a model with Cobb-Douglas preferences?

CES Utility

- The utility function may be stated as:

$$(x_1,\dots,x_n) = \alpha\left[\sum_{i=1}^{n} \beta_i x_i^{-\rho}\right]^{-1/\rho}$$

$$= \alpha\left[\beta_1 x_1^{-\rho} + \beta_2 x_2^{-\rho} + \cdots + \beta_n x_n^{-\rho}\right]^{-1/\rho}$$

- Without loss of generality, one can assume that α=1 and that the β_i sum to 1. (Why?)

CES Utility (Cont'd.)

- Consumer behavior with CES utility:
  - Strictly positive prices result in consumer demands
    - that are strictly positive for each good,
    - that exhaust income, and
    - that are continuously differentiable with respect to changes in relative prices.
  - What happens if prices are not strictly positive?

CES Utility (Cont'd.)

- Notice that the properties of the CES sound a lot like the Cobb-Douglas. Why?
- Recall the special cases of the CES:

	ρ	σ
Elastic	>0	>1
Inelastic	>-1,<0	<1

where σ is the pair-wise elasticity of substitution.

Benchmarking CES Utility

- Typically, a value for σ [= 1/(ρ-1), or ρ=(1- σ)/ σ] is obtained from the literature, α is set to 1, and the β's are assumed to sum to 1.
- For this case, the solution to the FOC's is to set:

$$\beta_i = \frac{p_i}{p_1}\left(\frac{x_i}{x_1}\right)^{1/\sigma}\left[\sum_{j=1}^{n}\frac{p_j}{p_1}\left(\frac{x_j}{x_1}\right)^{1/\sigma}\right]^{-1}$$

Benchmarking the CES (Cont'd.)

- What happens to this expression if σ=1?
- For the 1994 consumer expenditure data, with the assumption that σ=0.9, the resulting parameter values are

$$\rho = 0.\overline{1}11, \alpha = 1, \beta^{Food\&Drink} = 0.141, \beta^{Apparel} = 0.044$$

$$\beta^{Housing} = 0.330, \beta^{Transport.} = 0.186, \beta^{Other} = 0.300$$

CES Utility

- What implementation precautions do you think should be employed for a model based on CES utility?

Quadratic/Generalized Quadratic

- The utility function may be stated as:

$$(x_1, \cdots, x_n) = \alpha_0 + \sum_{i=1}^{n} \alpha_i f(x_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} f(x_i) f(x_j)$$

- Without loss of generality, one can assume that α_0=0. (Why?)

Generalized Quadratic (Cont'd.)

- Consumer behavior with Generalized Quadratic utility:
  - Utility may not be concave -- hence corner solutions can occur.
  - Even in the concave case, utility does not increase indefinitely -- hence the budget constraint may not be binding.
  - Demands are continuous (in the concave case) with respect to price changes.

Benchmarking Generalized Quadratics

- There are too many free parameters with this functional form -- the only workable strategy is as follows:
  - Specify the own-price and cross-price substitution effects exogenously (this specifies the β_{ij}'s which we hope form a negative definite matrix)
  - Use the FOC's to set the linear terms in the utility function so that observed quantities are generated by observed prices

Benchmarking Quadratics (Cont'd.)

- Other approaches involve assuming structure in the substitution relationships such as zero cross-price elasticities. (In the quadratic case, this amounts to assuming that the matrix [β_{ij}] is diagonal.)
  - In this case, utility can be independently benchmarked, one good at a time.
  - Let us benchmark for the zero cross-price elasticity case for the 1994 data for Food&Drink.

Benchmarking Quadratics (Cont'd.)

- The quantity and price data from our 1994 data are x = 4,689, and p = 1, and the relevant FOC is:

$$\partial U(\bar{x}) = \alpha + \beta \quad = \alpha + \beta \qquad =$$

- If the own-price elasticity is given to be -0.9, then we set β by solving the above for x, differentiating and manipulating as follows:

Benchmarking Quadratics (Cont'd.)

$$x = \frac{p}{\beta} - \frac{\alpha}{\beta} \quad \text{and} \quad \frac{\partial x}{\partial p} = \frac{1}{\beta} \quad \text{and} \quad \sigma = \frac{p}{x}\frac{\partial x}{\partial p} = \frac{p}{\beta x},$$

- or in the case of Food&Drink,

$$-0.9 = \frac{1}{\beta 4689} \quad or \quad \beta = -\frac{1}{0.9 \times 4689} = -2.37E-4$$

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Benchmarking Quadratics (Cont'd.)

- The final step in our benchmarking process is to compute the linear term based on the computed value for the quadratic term and the FOC.
- These procedures are roughly the same with the generalized quadratics although some modifications are needed. (In the more general case, econometrics is more commonly used.)

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