## Games and Market Imperfections

- Q: The mixed complementarity (MCP) framework is effective for modeling perfect markets, but can it handle imperfect markets?
- A: At least part of the time
- A particular type of game/market structure that can be handled as an MCP is the Nash equilibrium



## Nash Equilibria

- The description of agent behavior in a Nash setting sounds a lot like our view of an oligopolistic market for a homogeneous good
  - Agents make their choices knowing what impact their choices will have on the system *except* that they assume their choices will have no effect on the choices of others
  - In the example on endogenous price models, the oligopolistic producer chose its output level based on knowledge of consumers' response, but assuming that *other producers will not change*

## Nash Equilibrium (cont'd.)

- The key in the example we saw earlier was the symmetry of the players
  - This symmetry allowed us to express total market production as a function of individual production ($Q=ny$)
  - If symmetry is eliminated, the determination of the optimal production pattern is a difficult analytical problem and, in general, more easily attacked numerically



## Example – Electricity Markets

- Electricity is an instance of a market in the process of deregulation that has substantial opportunities for exercise of market power on the part of generators
  - Demand is inelastic
  - Due to costs of import/export, local generators have a competitive advantage
  - There are generally very few local generators serving a regional market

## Example – Electricity Markets

- Perfectly competitive models of electricity markets are not consistent with observed firm behavior – prices charged are far above marginal costs
- Essential model features:
  - Spatial disaggregation – demand, generation, inter-regional flows of electricity
  - Market power on the part of generators



## Example – Electricity Markets

- Simplest model – Agents
  - Consumers – described by a linear demand function
  - Producers – described by cost structure and capacity for generation, costs for inter-regional transmission, location of generation facilities, and Nash behavior

## Example – Electricity Markets

- Consumers – one for each region maximizing consumer's surplus

$$\underset{q_i}{\text{minimize}} - a_i q_i - b_i q_i^2 / 2 + \lambda_i q_i$$

- where $a_i$ and $b_i$ are the intercept and slope of the inverse demand function, $q_i$ is market demand in market $i$, and lambda in market $i$ is the price of electricity

- FOC's: $-a_i - b_i q_i + \lambda_i \geq 0,\ q_i(-a_i - b_i q_i + \lambda_i) = 0$



## Example – Electricity Markets

- Producers – profit maximizers (Nash)

$$\underset{g_{ik}\, f_{ijk}}{\text{minimize}} \sum_{i=1}^{n} \left\{ [-a_i - b_i q_i] \left[ g_{ik} + \sum_{j=1}^{n} ( f_{jik} - f_{ijk} ) \right] + d_{ik} g_{ik} + \sum^{n} w_{ij} f_{ijk} \right\}$$

- where $g_{ik}$ is electricity generation in region $i$ by firm $k$, $f_{ijk}$ is transmission of electricity from region $i$ to region $j$ by firm $k$, $d_{ik}$ is the (constant) marginal cost of generation, and $w_{ij}$ is the transmission cost from $i$ to $j$

## Example – Electricity Markets

- First constraint, total flows from *i* to *j* cannot exceed residual capacity

$$\sum_{s=1}^{S} f_{ijs} \le Fmax_{ij} \qquad : \mu_{ij}$$

- Second constraint, generation cannot exceed generation capacity

$$g_{ik} \le Gmax_{ik} \qquad : \beta_{ik}$$

- and, non-negativity $g_{ik} \ge 0, \; f_{ijk} \ge 0$



## Example – Electricity Markets

- The first-order conditions for $g_{ik}$ are

$$g_{ik}\left[-a_i - b_i q_i - b_i\left(g_{ik} + \sum_{j=1}^{N}(f_{jik} - f_{ijk})\right) + d_{ik}\right]^{+} +$$

$$(G\max_{ik} - g_{ik})\left[-a_i - b_i q_i - b_i\left(g_{ik} + \sum^{N}(f_{jik} - f_{ijk})\right) + d_{ik}\right]^{-} = 0$$

- (Notice that the third term in the inequality accounts for the fact that generators are Nash players and know that their choices affect demand)

## Example – Electricity Markets

- The first-order conditions for $f_{ijk}$ are

$$a_i + b_i q_i + b_i\left( g_{ik} + \sum_{m=1}^{N}( f_{mik} - f_{imk})\right)$$
$$- a_j - b_j q_j + b_j\left( g_{jk} + \sum_{m=1}^{N}( f_{mjk} - f_{jmk})\right)$$
$$+ w_{ij} - \mu_{ij} \geq 0$$

  - (Again note that the third term on the first and second lines reflect the Nash assumption)



## Example – Electricity Markets

- And,

$$f_{ijk}\left\{ a_i + b_i q_i + b_i\left( g_{ik} + \sum_{m=1}^{N}( f_{mik} - f_{imk})\right)\right.$$
$$\left. - a_j - b_j q_j - b_j\left( g_{jk} + \sum_{m=1}^{N}( f_{mjk} - f_{jmk})\right)\right.$$

- (Also note that the sum of out-shipments minus in-shipments generates lines one and two)

## Example – Electricity Markets

- The GAMS formulation of this problem:

```
set   k Firm index         / 1*3 /
      i Location index     / 1*4 /  ;
alias (k,s),(i,j,m,n) ;

parameter a(i)  Inverse demand intercept
      /1  520 ,  2  500 ,  3  500 ,  4  450 /;
parameter b(i)  Inverse demand slope
      /1  -1 ,  2  -1 ,  3  -1 ,  4  -1 /;
```



## Example – Electricity Markets

- Data for firms

```
table d(i,s)   Cost function linear term
      1      2      3
1     7.76
2            8.37   8.27
3     8.134
4                   7.94 ;
```

## Example – Electricity Markets

- table gmax(i,s) Generation maximum

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 530 | | |
| 2 | | 500 | 500 |
| 3 | 212 | | |
| 4 | | | 300 ; |



## Example – Electricity Markets

- Costs and limits relating to the transmission system

  - table w(i,j) Transmission charges

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | 1 | | 1.2 |
| 2 | 1 | | 1.4 | 1.05 |
| 3 | | 1.4 | | |
| 4 | 1.2 | 1.05 | | ; |

## Example – Electricity Markets

- table fmax(i,j) Maximum flow on line

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 80 | 120 | 120 |
| 2 | 80 | 0 | 140 | 150 |
| 3 | 120 | 140 | 0 | 0 |
| 4 | 120 | 150 | 0 | 0 ; |



## Example – Electricity Markets

- Next declare the variables
  - positive variables
  - g(i,s) Power generation by firm s at location i
  - f(i,j,s) Power flow from i to j generated by firm s
  - q(i) Power demand at i
  - lambda(i) Market price of power in i ;

## Example – Electricity Markets

- Notice that the maximum flow constraints from *i* to *j* are less than or equal to constraints – hence, the associated Lagrange multipliers should be non-positive
  - negative variables
  - mu(i,j) Shadow price on maximum flow from i to j;
- Don't forget the limits on generation capacity by region and firm
  - g.up(i,s) = gmax(i,s) ;



## Example – Electricity Markets

- Now declare equations
  - equations
  - dq(i) First-order conditions for q at location i
  - dg(i,s) First-order conditions for g
  - df(i,j,s) First-order conditions for f
  - sd(i) Supply-demand by firm and location
  - maxf(i,j) Maximum flow from i to j ;

## Example – Electricity Markets

- Now define the structure of the equations in GAMS notation:
  - First-order conditions for the consumer's problem
  - dq(i) ..
  -     -a(i)-b(i)*q(i)+lambda(i) =g= 0 ;



## Example – Electricity Markets

- First-order conditions for producer *s*'s choice of generation level in region *i*
  - dg(i,s)$d(i,s) ..
  -     -a(i)-b(i)*q(i)-b(i)*(g(i,s)
  -     +sum(j,f(j,i,s)$d(j,s)-f(i,j,s)$d(i,s)))
  -     +d(i,s) =g= 0 ;

## Example – Electricity Markets

- First-order conditions for producer *s*'s choice of transmission level from region *i* to region *j*

```
df(i,j,s)$d(i,s) ..
   a(i)+b(i)*q(i)+b(i)*(g(i,s)
   +sum(m,f(m,i,s)$d(m,s)-f(i,m,s)$d(i,s)))
   -a(j)-b(j)*q(j)-b(j)*(g(j,s)
   +sum(m,f(j,m,s)$d(j,s)-f(m,j,s)$d(m,s)))
   +w(i,j)-mu(i,j) =g= 0 ;
```



## Example – Electricity Markets

- Supply demand balance for region *i*

```
sd(i) ..
   sum(s,g(i,s)$d(i,s)
   - sum(j,f(i,j,s)$d(i,s)-f(j,i,s)$d(j,s)))
   =g= q(i) ;
```

- Flow limits from *i* to *j*

```
maxf(i,j) ..
   sum(s$d(i,s),f(i,j,s)) =l= fmax(i,j) ;
```

## Example – Electricity Markets

- Model and solve statements
  - model nash / dq.q,dg.g,df.f,sd.lambda,maxf.mu / ;
  - solve nash using mcp ;



## Example – Electricity Markets

- Solution output:

---- EQU dq First-order conditions for q at location i

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1 | 520.000 | 520.000 | +INF | 356.120 |
| 2 | 500.000 | 500.000 | +INF | 401.120 |
| 3 | 500.000 | 500.000 | +INF | 212.000 |
| 4 | 450.000 | 450.000 | +INF | 356.030 |

## Example – Electricity Markets

- ---- EQU dg First-order conditions for g

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1.1 | 512.240 | 512.240 | +INF | 356.120 |
| 2.2 | 491.630 | 491.630 | +INF | 380.510 |
| 2.3 | 491.730 | 491.730 | +INF | 170.610 |
| 3.1 | 491.866 | 284.000 | +INF | 212.000 REDEF |
| 4.3 | 442.060 | 442.060 | +INF | 206.030 |

- (Notice the "REDEF" indication for g(3,1) – this relates to the upper bounds on g with dg as a $\leq$)



## Example – Electricity Markets

- ---- EQU df First-order conditions for f

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1.1.1 | . | 400.000 | +INF | . |
| 1.2.1 | -21.000 | -331.120 | +INF | 80.000 REDEF |
| 1.3.1 | -20.000 | 51.760 | +INF | . |
| 1.4.1 | -71.200 | -276.210 | +INF | 120.000 REDEF |
| 2.1.2 | 19.000 | 19.015 | +INF | . |
| 2.1.3 | 19.000 | -181.085 | +INF | 80.000 REDEF |
| 2.2.2 | . | 580.000 | +INF | . |
| 2.2.3 | . | 160.000 | +INF | . |
| 2.3.2 | -1.400 | -141.295 | +INF | 140.000 REDEF |
| 2.3.3 | -1.400 | -1.395 | +INF | . |

## Example – Electricity Markets

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 2.4.2 | -51.050 | -285.600 | +INF | 150.000 REDEF |
| 2.4.3 | -51.050 | 190.330 | +INF | . |
| 3.1.1 | 20.000 | 628.240 | +INF | . |
| 3.2.1 | -1.400 | -102.880 | +INF | 140.000 REDEF |
| 3.3.1 | . | 280.000 | +INF | . |
| 3.4.1 | -50.000 | -47.970 | +INF | . |
| 4.1.3 | 68.800 | -285.940 | +INF | 120.000 REDEF |
| 4.2.3 | 48.950 | 209.670 | +INF | . |
| 4.3.3 | 50.000 | -230.060 | +INF | . |
| 4.4.3 | . | 240.000 | +INF | . |



## Example – Electricity Markets

---- EQU sd Supply-demand balance location

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1 | . | . | +INF | 163.880 |
| 2 | . | . | +INF | 98.880 |
| 3 | . | . | +INF | 288.000 |
| 4 | . | . | +INF | 93.970 |

## Example – Electricity Markets

---- EQU maxf Maximum flow from i to j

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1.1 | -INF | . | . | . |
| 1.2 | -INF | 80.000 | 80.000 | . |
| 1.3 | -INF | . | 120.000 | . |
| 1.4 | -INF | 120.000 | 120.000 | . |
| 2.1 | -INF | 80.000 | 80.000 | -154.525 |
| 2.2 | -INF | . | . | . |
| 2.3 | -INF | 140.000 | 140.000 | -278.335 |
| 2.4 | -INF | 150.000 | 150.000 | . |



## Example – Electricity Markets

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 3.1 | -INF | . | 120.000 | . |
| 3.2 | -INF | 140.000 | 140.000 | . |
| 3.3 | -INF | . | . | . |
| 3.4 | -INF | . | . | . |
| 4.1 | -INF | 120.000 | 120.000 | . |
| 4.2 | -INF | . | 150.000 | . |
| 4.3 | -INF | . | . | . |
| 4.4 | -INF | . | . | . |

## Example – Electricity Markets

---- VAR g Power generation by firm s at location i

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1.1 | . | 356.120 | 530.000 | . |
| 1.2 | . | . | . | EPS |
| 1.3 | . | . | . | EPS |
| 2.1 | . | . | . | EPS |
| 2.2 | . | 380.510 | 500.000 | . |
| 2.3 | . | 170.610 | 500.000 | . |
| 3.1 | . | 212.000 | 212.000 | -207.866 |
| 3.2 | . | . | . | EPS |
| 3.3 | . | . | . | EPS |
| 4.1 | . | . | . | EPS |
| 4.2 | . | . | . | EPS |
| 4.3 | . | 206.030 | 300.000 | . |



## Example – Electricity Markets

---- VAR f Power flow from location i to j generated by firm s

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1.1.1 | . | . | . | 400.000 |
| 1.2.1 | . | 80.000 | 80.000 | -310.120 |
| 1.3.1 | . | . | 120.000 | 71.760 |
| 1.4.1 | . | 120.000 | 120.000 | -205.010 |
| 2.1.2 | . | . | 80.000 | 0.015 |
| 2.1.3 | . | 80.000 | 80.000 | -200.085 |
| 2.2.2 | . | . | . | 580.000 |
| 2.2.3 | . | . | . | 160.000 |
| 2.3.2 | . | 140.000 | 140.000 | -139.895 |
| 2.3.3 | . | . | 140.000 | 0.005 |
| 2.4.2 | . | 150.000 | 150.000 | -234.550 |

## Example – Electricity Markets

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 2.4.3 | . | . | 150.000 | 241.380 |
| 3.1.1 | . | . | 120.000 | 608.240 |
| 3.2.1 | . | 140.000 | 140.000 | -101.480 |
| 3.3.1 | . | . | . | 280.000 |
| 3.4.1 | . | . | . | 2.030 |
| 4.1.3 | . | 120.000 | 120.000 | -354.740 |
| 4.2.3 | . | . | 150.000 | 160.720 |
| 4.3.3 | . | . | . | -280.060 |
| 4.4.3 | . | . | . | 240.000 |

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## Example – Electricity Markets

---- VAR q Power demand at location i

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 1 | . | 356.120 | +INF | . |
| 2 | . | 401.120 | +INF | . |
| 3 | . | 212.000 | +INF | . |
| 4 | . | 356.030 | +INF | . |

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## Example – Electricity Markets

l ---- VAR lambda Market price in region i

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| l 1 | . | 163.880 | +INF | . |
| l 2 | . | 98.880 | +INF | . |
| l 3 | . | 288.000 | +INF | . |
| l 4 | . | 93.970 | +INF | . |



## Example – Electricity Markets

l ---- VAR mu Shadow price on max flow i to j

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| l 1.1 | -INF | . | . | EPS |
| l 1.2 | -INF | . | . | EPS |
| l 1.3 | -INF | . | . | -120.000 |
| l 1.4 | -INF | . | . | EPS |
| l 2.1 | -INF | -154.525 | . | . |
| l 2.2 | -INF | . | . | EPS |
| l 2.3 | -INF | -278.335 | . | . |
| l 2.4 | -INF | . | . | EPS |

## Example – Electricity Markets

| | LOWER | LEVEL | UPPER | MARGINAL |
|---|---|---|---|---|
| 3.1 | -INF | . | . | -120.000 |
| 3.2 | -INF | . | . | EPS |
| 3.3 | -INF | . | . | EPS |
| 3.4 | -INF | . | . | EPS |
| 4.1 | -INF | . | . | EPS |
| 4.2 | -INF | . | . | -150.000 |
| 4.3 | -INF | . | . | EPS |
| 4.4 | -INF | . | . | EPS |



## Example – Electricity Markets

- The MCP framework can also be used to formulate certain types of
  - Games
  - Imperfect markets
- It cannot be used for problems with
  - Indivisibilities, and
  - Uniqueness of solutions can be difficult to establish