Endogenous Price Models

- These models allow us to impose relationships between primal and dual solutions of our models
- This is important when the model is representing a large enough portion of a market
  - A LARGE number of identical producers
  - A number of Large producers

Endogenous Price Models (cont'd.)

- Recall the case of a single producer, single consumer endogenous price model:

$$\underset{S,D}{\text{maximize}} \int_0^D (d/a)^{1/b}\, dd - \int_0^S (s/c)^{1/d}\, ds$$

- While it says "single" producer/consumer, the view is really that there is an extremely large number of producers and consumers that are treated as single aggregate decision making units

Endogenous Price Models (cont'd.)

- The model above is stated for a single good at a single location
- This can be generalized to multiple goods and locations
- These generalizations are addressed in the book: Spatial and Temporal Price and Allocaiton Models, by Takayama and Judge (1971)

Endogenous Price Models (cont'd.)

- This type of model has fallen somewhat out of favor in programming circles because they are difficult to calibrate (benchmark)
- Equilibrium modeling has risen as the replacement
- Before we look closely at equilibrium modeling, it may help to understand the endogenous price modeling approach in cases where there is market power

Endogenous Price Models (cont'd.)

- Consider the formulation of models representing the spectrum of market power:
  - Perfect competition
  - Oligopoly
  - Monopoly
- The first and last of these are relatively straightforward

Endogenous Price Models (cont'd.)

- Framework:
  - Single commodity and location
  - Demand relationship $D = a - bp$ or $p = (a - D)/b$
  - Production relationship $y = f(x)$ where x is a vector of inputs available at fixed costs defined by the vector c

Endogenous Price Models (cont'd.)

- All producer's behavior can be described by the following problem:

$$\underset{y,x}{\text{maximize}}\ py - c^t x$$
$$\text{subject to}: y = f(x)$$

- where it is assumed for convenience that the marginal physical product of each input approaches infinity as the level of the input approaches zero
- (Assume an interior solution)

Endogenous Price Models (cont'd.)

- The difference between our perfect competitor, oligopolist, and monopolist lies entirely in how the view that their choices will affect the market price, *p*
  - The perfect competitor assume that market price is not affected by their choices
  - The monopolist knows that her choices *determine* market price
  - The oligopolist knows that his choices influence market price *and* what the other producers choose

Endogenous Price Models (cont'd.)

- So, all of these producers are working with the same Lagrangian:

$$L(y,x,\pi) = py - c^t x - \pi[y - f(x)]$$

- The difference shows up when we take derivatives of the Lagrangian

Endogenous Price Models (cont'd.)

- Perfect Competition – in this case the derivative of price with respect to output is zero, and the FOC's are:

$$\frac{\partial L(y,x,\pi)}{\partial y} = p - \pi = 0$$

$$\frac{\partial L(y,x,\pi)}{\partial x_i} = -c_i + \pi \frac{\partial f(x)}{\partial x_i} = 0$$

Endogenous Price Models (cont'd.)

- Focusing on the first of these and substituting our relationship for price and total demand quantity yields

$$(a-y)/b-\pi=0 \text{ or } \pi=(a-y)/b$$

- However, what we need is a math program that will incorporate this "side condition" relating price and total demand

Endogenous Price Models (cont'd.)

- The problem that does this is:

$$\underset{y,x}{\text{maximize}}\ [a-y/(2b)]y-c^{t}x$$

- Note that the revenue term here is replaced by the integral under the inverse demand curve and has a "quasi-welfare" interpretation

Endogenous Price Models (cont'd.)

- Monopolistic Case – in this case the producer "sees the slope of the demand curve", and the derivative of revenue with respect to quantity is as follows:

$$\frac{\partial py}{\partial y} = \frac{\partial[y(a-y)/b]}{\partial y} = (a-2y)/b$$
$$= p - y/b$$

Endogenous Price Models (cont'd.)

- So, the FOC's in the monopolistic case are:

$$\frac{\partial L(y,x,\pi)}{\partial y} = (a-2y)/b - \pi = 0$$
$$\frac{\partial L(y,x,\pi)}{\partial x_i} = -c_i + \pi \frac{\partial f(x)}{\partial x_i} = 0$$

Endogenous Price Models (cont'd.)

- Focusing on the first of these we find that the relationship between shadow price and total supply is now different

$$(a - 2y)/b - \pi = 0 \text{ or } y = (a - b\pi)/2$$

- Notice that due to linearity of the demand curve, supply is half of the competitive case

Endogenous Price Models (cont'd.)

- The problem that does this is:

$$\underset{y,x}{\text{maximize}}\ [(a - y)/b]y - c^t x$$

- Note that the revenue term here is total revenue and reflects the slope of the inverse demand curve

Endogenous Price Models (cont'd.)

- Oligopolistic Case (*n* Identical Producers) – in this case the producer knows that his choice affects price and the behavior of other producers
- Total supply is *n* times individual supply ($D = ny$), and the price relationship is $p = (a - (n-1)/nD - y)/b$
- This means:

$$\frac{\partial py}{\partial y} = \left(a - \frac{n-1}{n}D - 2y\right)/b = [a-(n+1)y]/b = p - y/b$$

Endogenous Price Models (cont'd.)

- So, the FOC's for this problem are:

$$\frac{\partial L(y,x,\pi)}{\partial y} = [a-(n+1)y]/b - \pi = 0$$

$$\frac{\partial L(y,x,\pi)}{\partial x_i} = -c_i + \pi\frac{\partial f(x)}{\partial x_i} = 0$$

Endogenous Price Models (cont'd.)

- Again focusing on the first of these and rewriting it to get output as a function of shadow price yields

$$a-(n+1)y]/b-\pi=0$$
$$\text{or } y=(a-\pi b)/(n+1)$$

- And industry supply is $D=(a-b\pi)n/(n+1)$

- Again, what we need is a math program that has a Lagrangian consistent with the above FOC's

Endogenous Price Models (cont'd.)

- The problem that does this is:

$$\underset{y,x}{\text{maximize}}\, [a-(n+1)y/(2b)]y-c^{t}x$$

- Note that the revenue term here is total revenue and reflects the slope of the inverse demand curve

Endogenous Price Models (cont'd.)

- Where do we go from here?
- Where have we been?
  - All models have had a single agent perspective
  - If there were multiple agents (e.g., multiple producers) they were all identical, or
  - Their goals were compatible (e.g., producers serving the needs of consumers)

Endogenous Price Models (cont'd.)

- The next step is to consider multiple agents who are either:
  - Not identical (e.g., asymmetric costs or demands) and/or
  - That have conflicting objectives
- Models in this class include equilibrium models and some types of games