Introduction to GAMS

An Example of a Transportation Problem

Example – A Transportation Problem

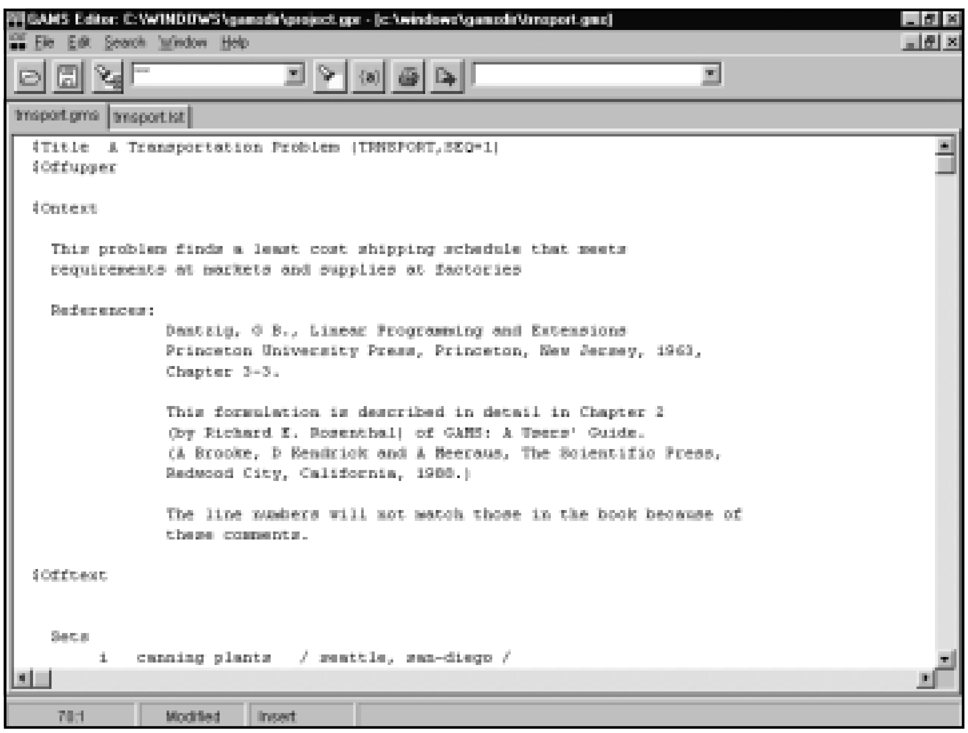
- $$\underset{z,x_{ij}}{\text{maximize}}\ z = \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij}$$

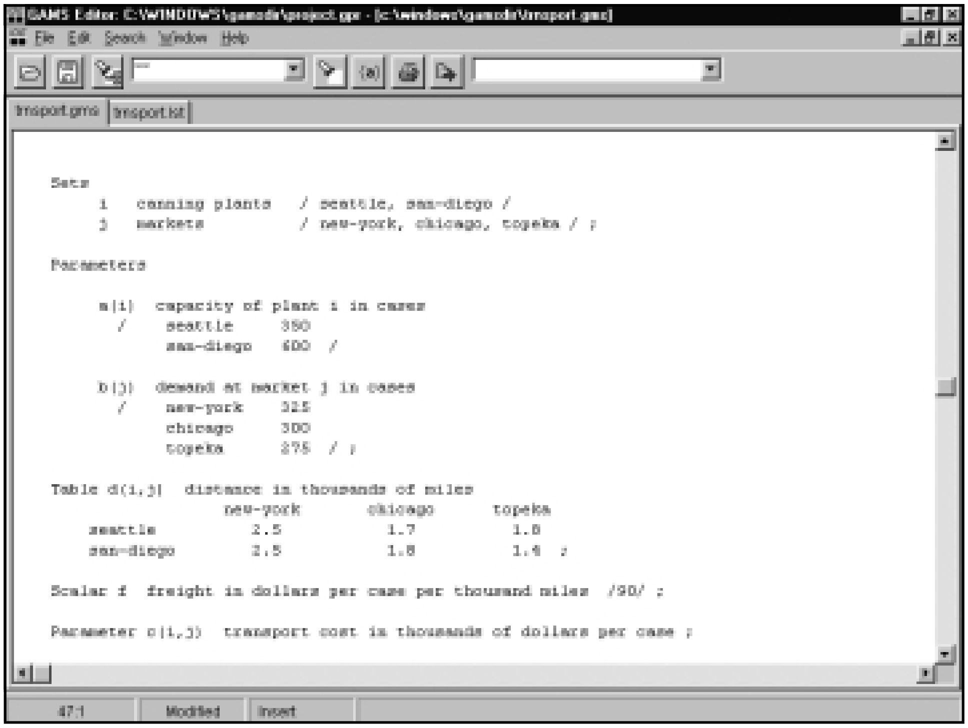
  subject to:

$$\sum_{i=1}^{n} x_{ij} \geq b_j \qquad j = 1,\dots,m$$

$$\sum_{j=1}^{m} x_{ij} \leq a_i \qquad i = 1,\dots,n$$

Video Goes Here

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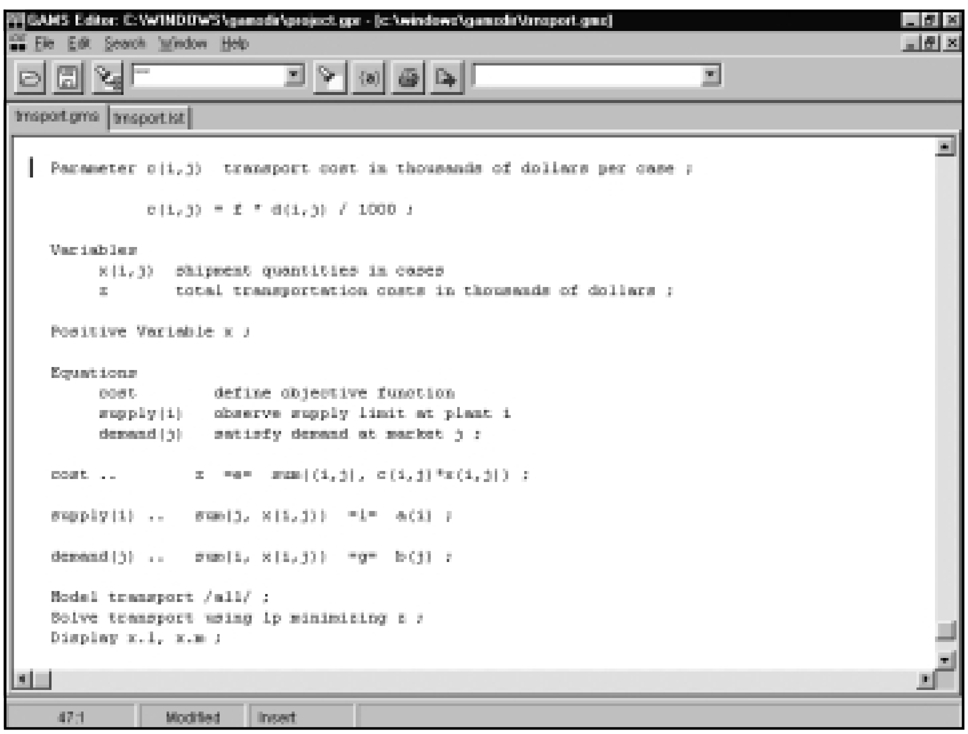
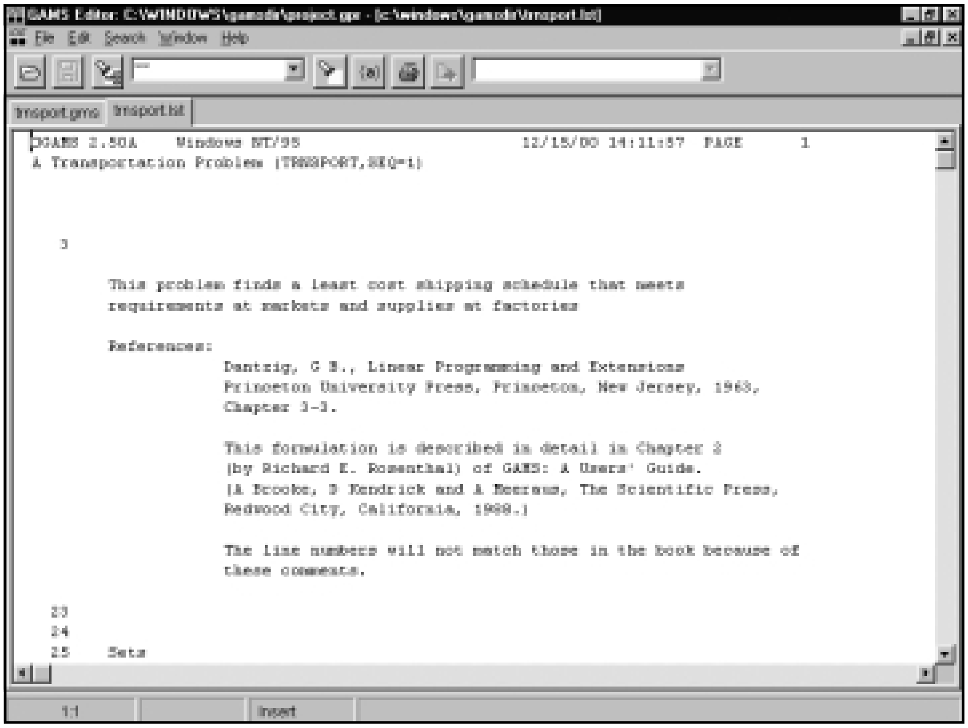
File Edit Search Window Help
trnsport.gms
trnsport.lst
Sets
i canning plants / seattle, san-diego /
j markets / new-york, chicago, topeka / ;
Parameters
a(i) capacity of plant i in cases
/ seattle 350
san-diego 600 /
b(j) demand at market j in cases
/ new-york 325
chicago 300
topeka 275 / ;
Table d(i,j) distance in thousands of miles
new-york chicago topeka
seattle 2.5 1.7 1.8
san-diego 2.5 1.8 1.4 ;
Scalar f freight in dollars per case per thousand miles /90/ ;
Parameter c(i,j) transport cost in thousands of dollars per case ;
47:1
Modified
Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.gms]
File Edit Search Window Help
trnsport.gms
trnsport.lst
Parameter c(i,j) transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
x(i,j) shipment quantities in cases
z total transportation costs in thousands of dollars ;
Positive Variable x ;
Equations
cost define objective function
supply(i) observe supply limit at plant i
demand(j) satisfy demand at market j ;
cost .. z =e= sum((i,j), c(i,j)*x(i,j)) ;
supply(i) .. sum(j, x(i,j)) =l= a(i) ;
demand(j) .. sum(i, x(i,j)) =g= b(j) ;
Model transport /all/ ;
Solve transport using lp minimizing z ;
Display x.l, x.m ;
47:1
Modified
Insert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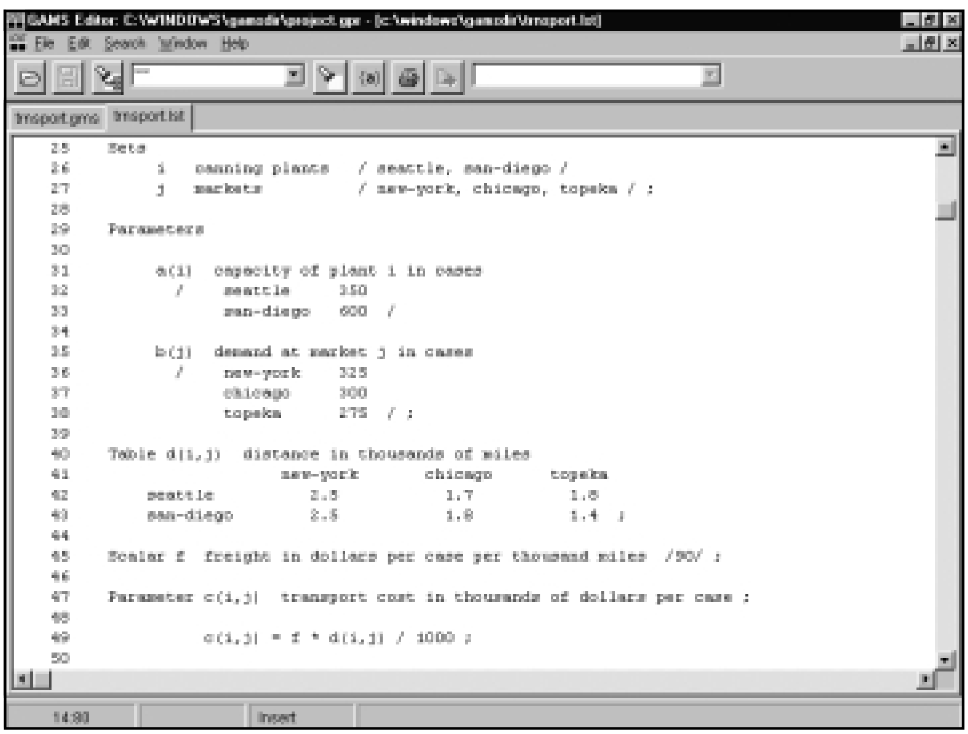
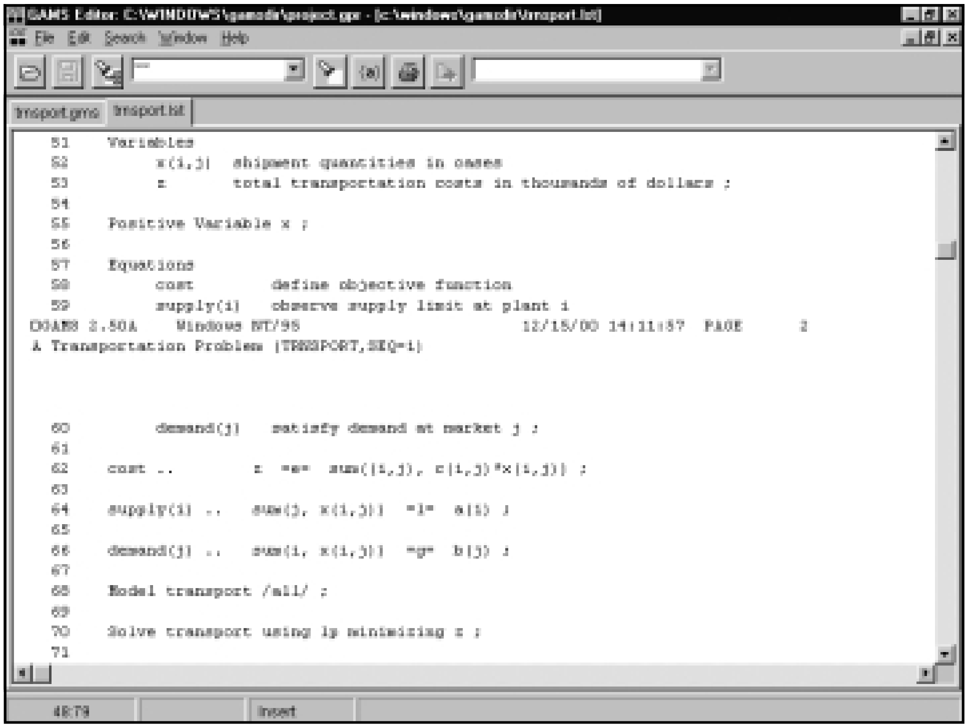
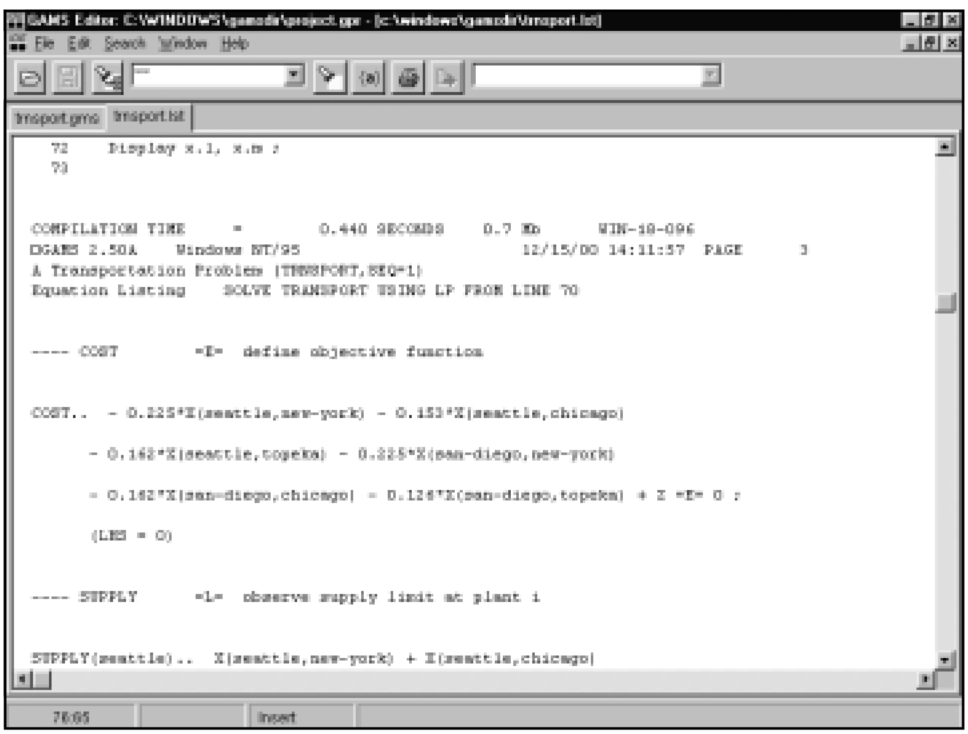
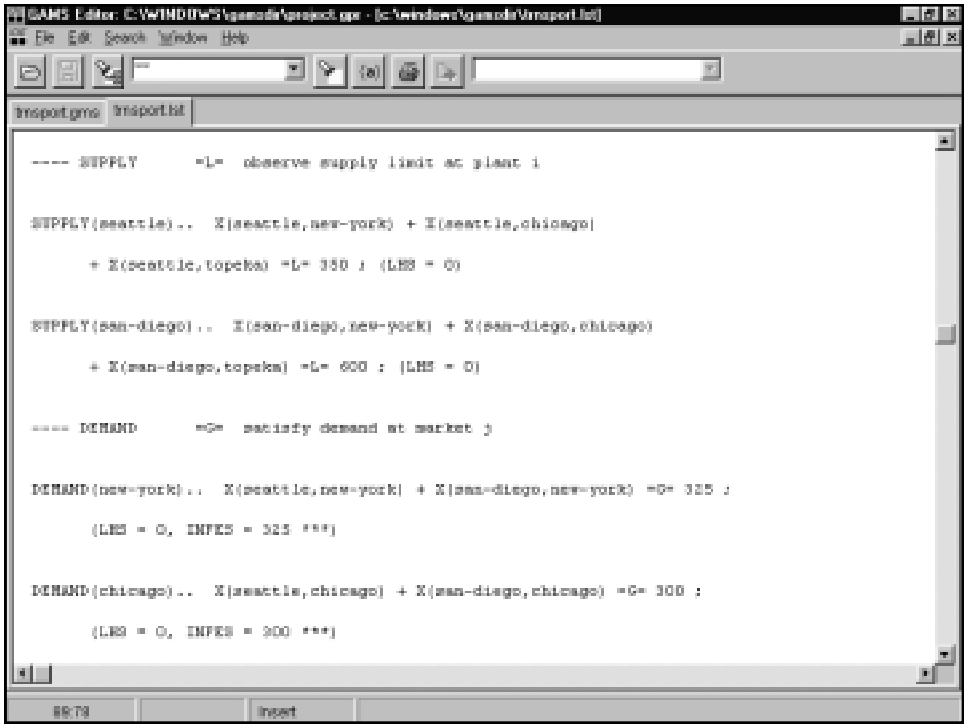
File Edit Search Window Help
trnsport.gms
trnsport.lst
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 1
A Transportation Problem (TRNSPORT,SEQ=1)
3
This problem finds a least cost shipping schedule that meets
requirements at markets and supplies at factories
References:
Dantzig, G B., Linear Programming and Extensions
Princeton University Press, Princeton, New Jersey, 1963,
Chapter 3-3.
This formulation is described in detail in Chapter 2
(by Richard E. Rosenthal) of GAMS: A Users' Guide.
(A Brooke, D Kendrick and A Meeraus, The Scientific Press,
Redwood City, California, 1988.)
The line numbers will not match those in the book because of
these comments.
23
24
25 Sets
1:1
Insert

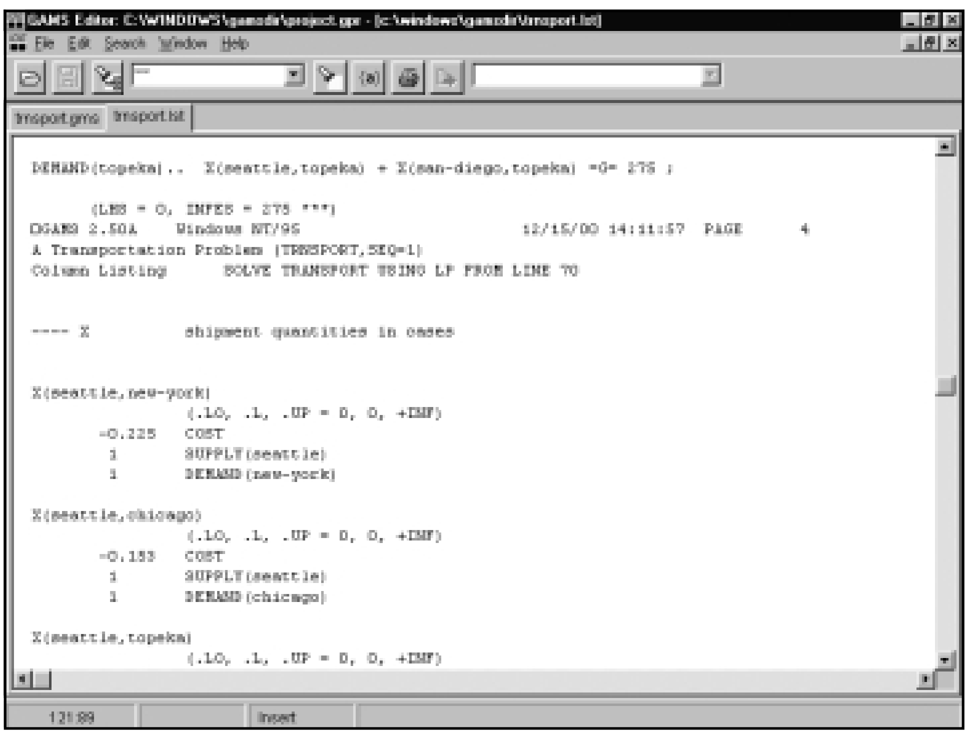

GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
25 Sets
26 i canning plants / seattle, san-diego /
27 j markets / new-york, chicago, topeka / ;
28
29 Parameters
30
31 a(i) capacity of plant i in cases
32 / seattle 350
33 san-diego 600 /
34
35 b(j) demand at market j in cases
36 / new-york 325
37 chicago 300
38 topeka 275 / ;
39
40 Table d(i,j) distance in thousands of miles
41 new-york chicago topeka
42 seattle 2.5 1.7 1.8
43 san-diego 2.5 1.8 1.4 ;
44
45 Scalar f freight in dollars per case per thousand miles /90/ ;
46
47 Parameter c(i,j) transport cost in thousands of dollars per case ;
48
49 c(i,j) = f * d(i,j) / 1000 ;
50
14:80
Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
51 Variables
52 x(i,j) shipment quantities in cases
53 z total transportation costs in thousands of dollars ;
54
55 Positive Variable x ;
56
57 Equations
58 cost define objective function
59 supply(i) observe supply limit at plant i
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 2
A Transportation Problem (TRNSPORT,SEQ=1)
60 demand(j) satisfy demand at market j ;
61
62 cost .. z =e= sum((i,j), c(i,j)*x(i,j)) ;
63
64 supply(i) .. sum(j, x(i,j)) =l= a(i) ;
65
66 demand(j) .. sum(i, x(i,j)) =g= b(j) ;
67
68 Model transport /all/ ;
69
70 Solve transport using lp minimizing z ;
71
48:79
Insert

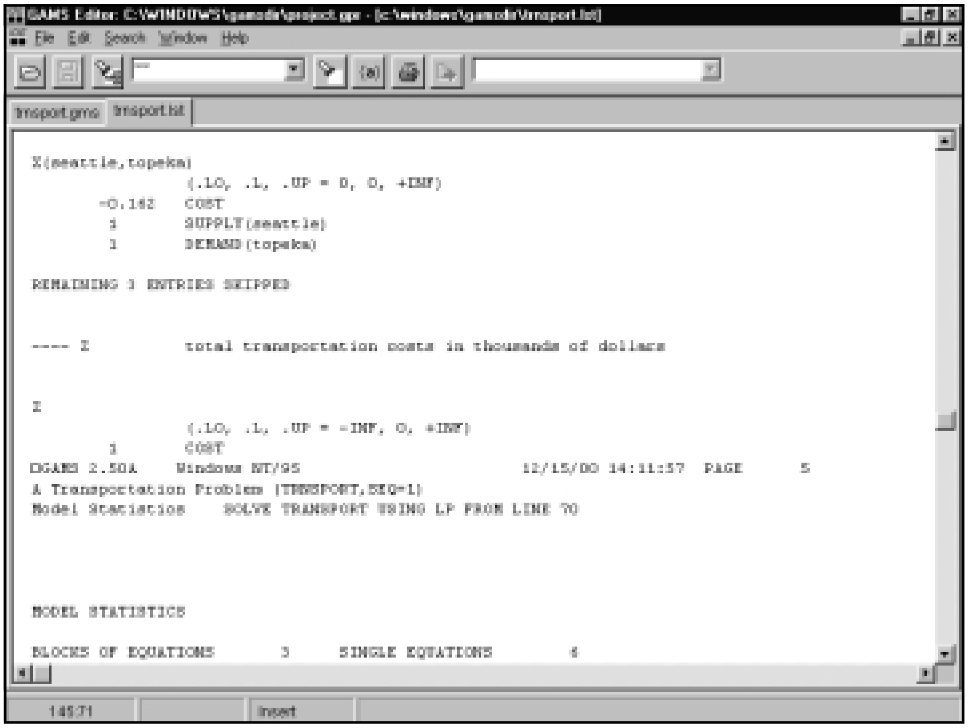

GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
72 Display x.l, x.m ;
73
COMPILATION TIME = 0.440 SECONDS 0.7 Mb WIN-18-096
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 3
A Transportation Problem (TRNSPORT,SEQ=1)
Equation Listing SOLVE TRANSPORT USING LP FROM LINE 70
---- COST =E= define objective function
COST.. - 0.225*X(seattle,new-york) - 0.153*X(seattle,chicago)
- 0.162*X(seattle,topeka) - 0.225*X(san-diego,new-york)
- 0.162*X(san-diego,chicago) - 0.126*X(san-diego,topeka) + Z =E= 0 ;
(LHS = 0)
---- SUPPLY =L= observe supply limit at plant i
SUPPLY(seattle).. X(seattle,new-york) + X(seattle,chicago)
76:65
Insert

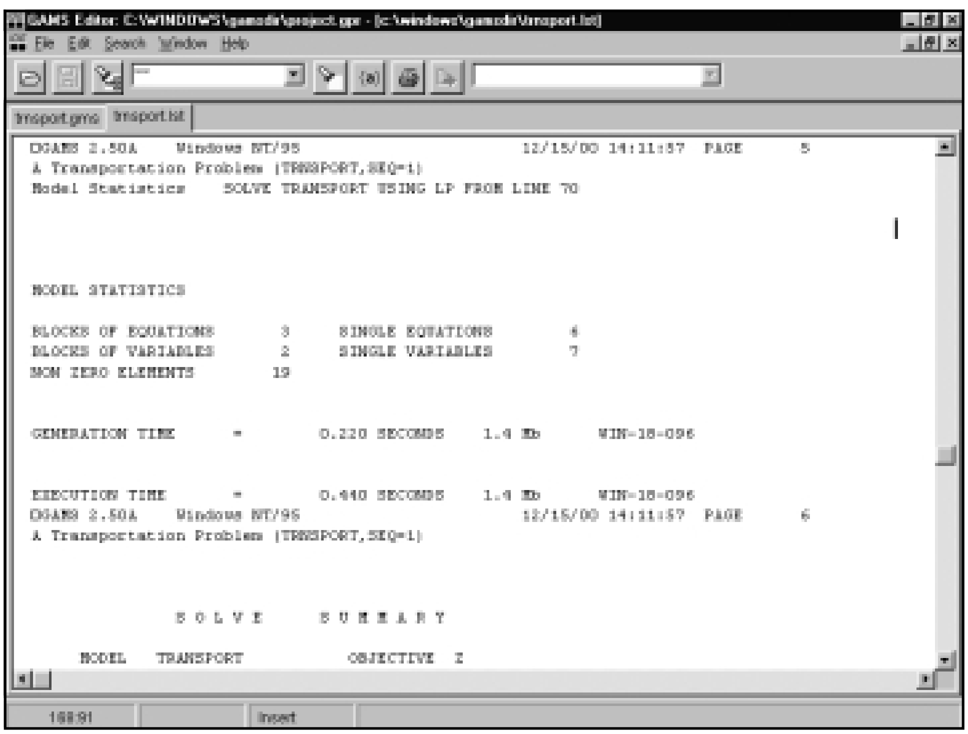


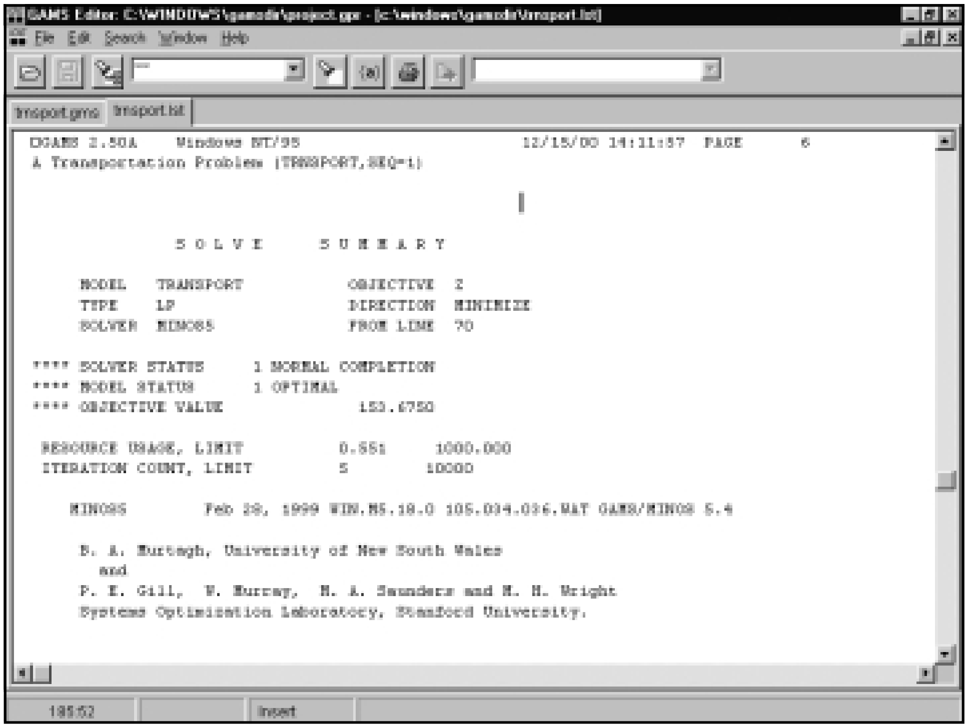
```
---- SUPPLY      =L=  observe supply limit at plant i

SUPPLY(seattle)..  X(seattle,new-york) + X(seattle,chicago)

      + X(seattle,topeka) =L= 350 ; (LHS = 0)

SUPPLY(san-diego)..  X(san-diego,new-york) + X(san-diego,chicago)

      + X(san-diego,topeka) =L= 600 ; (LHS = 0)

---- DEMAND      =G=  satisfy demand at market j

DEMAND(new-york)..  X(seattle,new-york) + X(san-diego,new-york) =G= 325 ;

      (LHS = 0, INFES = 325 ***)

DEMAND(chicago)..  X(seattle,chicago) + X(san-diego,chicago) =G= 300 ;

      (LHS = 0, INFES = 300 ***)
```

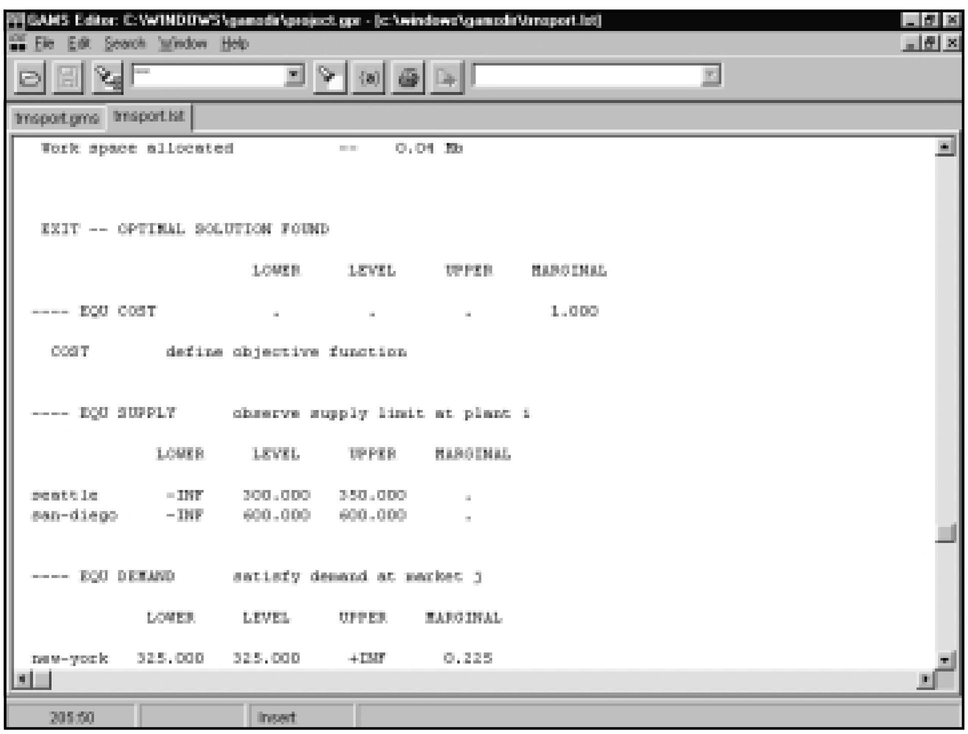


```
DEMAND(topeka)..  X(seattle,topeka) + X(san-diego,topeka) =G= 275 ;

      (LHS = 0, INFES = 275 ***)
GAMS 2.50A    Windows NT/95                    12/15/00 14:11:57  PAGE      4
A Transportation Problem (TRNSPORT,SEQ=1)
Column Listing      SOLVE TRANSPORT USING LP FROM LINE 70


---- X          shipment quantities in cases


X(seattle,new-york)
                (.LO, .L, .UP = 0, 0, +INF)
       -0.225   COST
        1       SUPPLY(seattle)
        1       DEMAND(new-york)

X(seattle,chicago)
                (.LO, .L, .UP = 0, 0, +INF)
       -0.153   COST
        1       SUPPLY(seattle)
        1       DEMAND(chicago)

X(seattle,topeka)
                (.LO, .L, .UP = 0, 0, +INF)
```


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms trnsport.lst
X(seattle,topeka)
(.LO, .L, .UP = 0, 0, +INF)
-0.162 COST
1 SUPPLY(seattle)
1 DEMAND(topeka)
REMAINING 3 ENTRIES SKIPPED
---- Z total transportation costs in thousands of dollars
Z
(.LO, .L, .UP = -INF, 0, +INF)
1 COST
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 5
A Transportation Problem (TRNSPORT,SEQ=1)
Model Statistics SOLVE TRANSPORT USING LP FROM LINE 70
MODEL STATISTICS
BLOCKS OF EQUATIONS 3 SINGLE EQUATIONS 6
145:71 Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms trnsport.lst
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 5
A Transportation Problem (TRNSPORT,SEQ=1)
Model Statistics SOLVE TRANSPORT USING LP FROM LINE 70
MODEL STATISTICS
BLOCKS OF EQUATIONS 3 SINGLE EQUATIONS 6
BLOCKS OF VARIABLES 2 SINGLE VARIABLES 7
NON ZERO ELEMENTS 19
GENERATION TIME = 0.220 SECONDS 1.4 Mb WIN-18-096
EXECUTION TIME = 0.440 SECONDS 1.4 Mb WIN-18-096
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 6
A Transportation Problem (TRNSPORT,SEQ=1)
S O L V E S U M M A R Y
MODEL TRANSPORT OBJECTIVE Z
168:91 Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
GAMS 2.50A Windows NT/95 12/15/00 14:11:37 PAGE 6
A Transportation Problem (TRNSPORT,SEQ=1)
S O L V E S U M M A R Y
MODEL TRANSPORT OBJECTIVE Z
TYPE LP DIRECTION MINIMIZE
SOLVER MINOS5 FROM LINE 70
**** SOLVER STATUS 1 NORMAL COMPLETION
**** MODEL STATUS 1 OPTIMAL
**** OBJECTIVE VALUE 153.6750
RESOURCE USAGE, LIMIT 0.551 1000.000
ITERATION COUNT, LIMIT 5 10000
MINOS5 Feb 28, 1999 WIN.M5.18.0 105.034.036.WAT GAMS/MINOS 5.4
B. A. Murtagh, University of New South Wales
and
P. E. Gill, W. Murray, M. A. Saunders and M. H. Wright
Systems Optimization Laboratory, Stanford University.
185:52
Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
Work space allocated -- 0.04 Mb
EXIT -- OPTIMAL SOLUTION FOUND
LOWER LEVEL UPPER MARGINAL
---- EQU COST . . . 1.000
COST define objective function
---- EQU SUPPLY observe supply limit at plant i
LOWER LEVEL UPPER MARGINAL
seattle -INF 300.000 350.000 .
san-diego -INF 600.000 600.000 .
---- EQU DEMAND satisfy demand at market j
LOWER LEVEL UPPER MARGINAL
new-york 325.000 325.000 +INF 0.225
205:50
Insert

GAMS View of a Problem

$$
\begin{aligned}
&\textit{optimize } z \\
&\textit{subject to}: \\
&\qquad z = f(x) \\
&\qquad \underline{g}_j \le g_j(x) \le \overline{g}_j \qquad j = 1,\ldots,m \\
&\qquad l_i \le x_i \le u_i \qquad\quad i = 1,\ldots,n
\end{aligned}
$$

Bounded Equations

Equation	Lower Bd.	Upper Bd.
$\le$	–INF	b
$\ge$	b	+INF
=	b	b

- Finite upper *and* lower bounds simultaneously can be accomodated

Shadow Prices/Penalty Costs

- Shadow prices are “marginals”

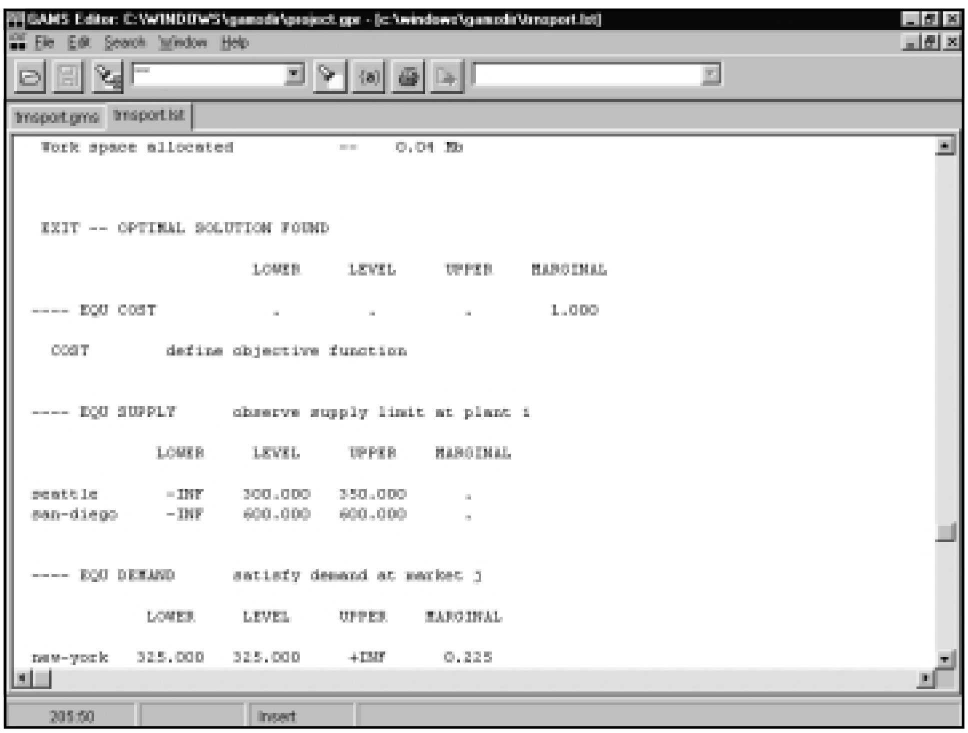
$$\textit{optimize}\ z$$

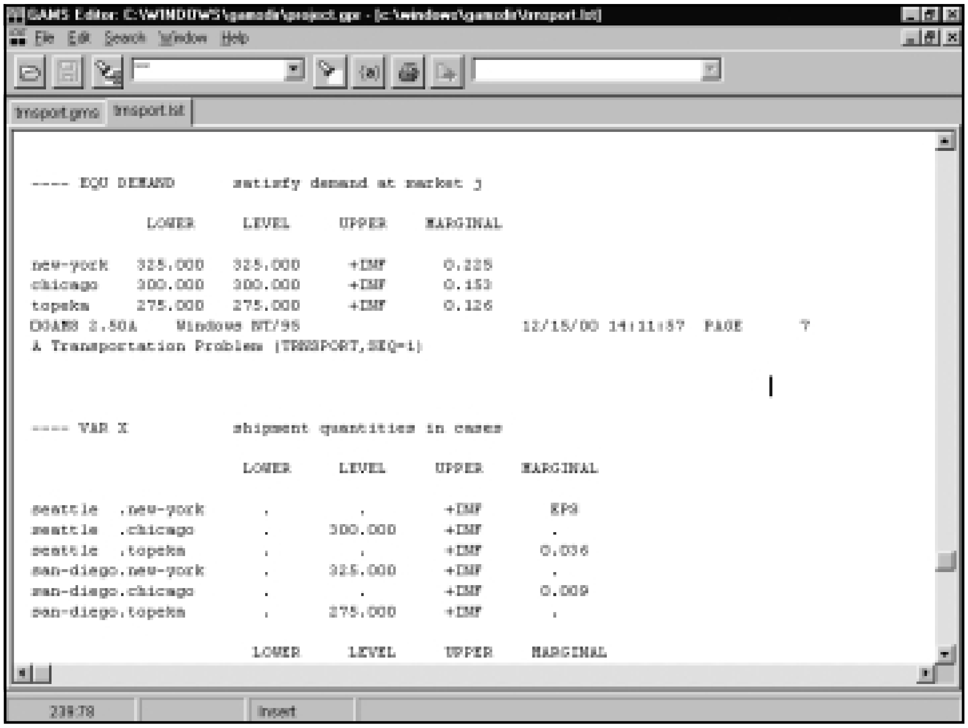
$$\textit{subject to}:$$

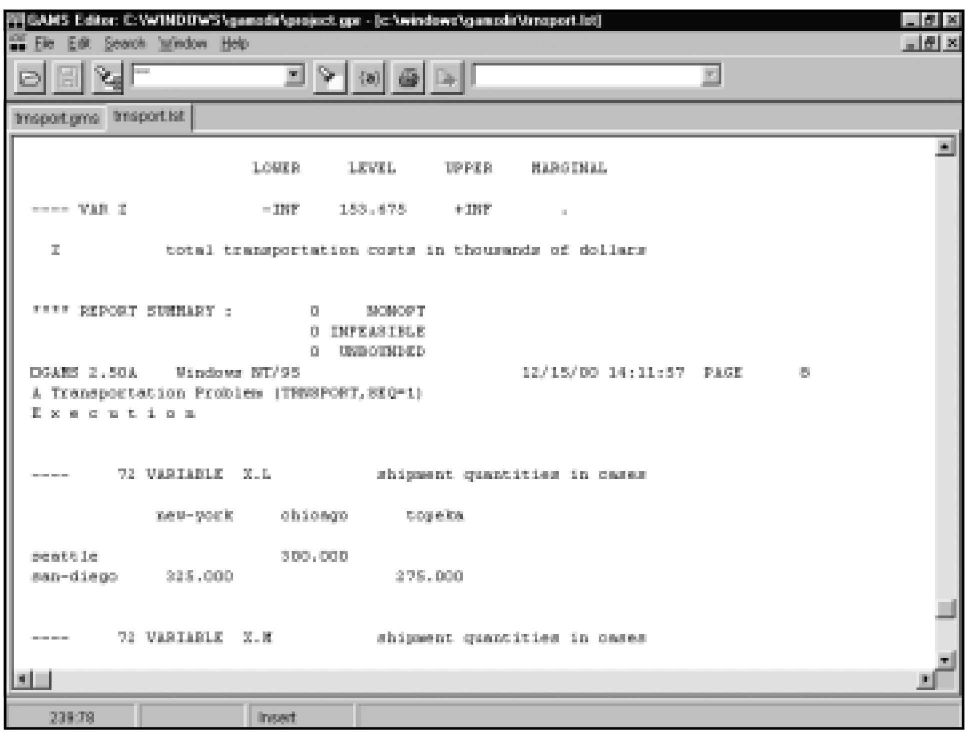
$$z = f(x)$$

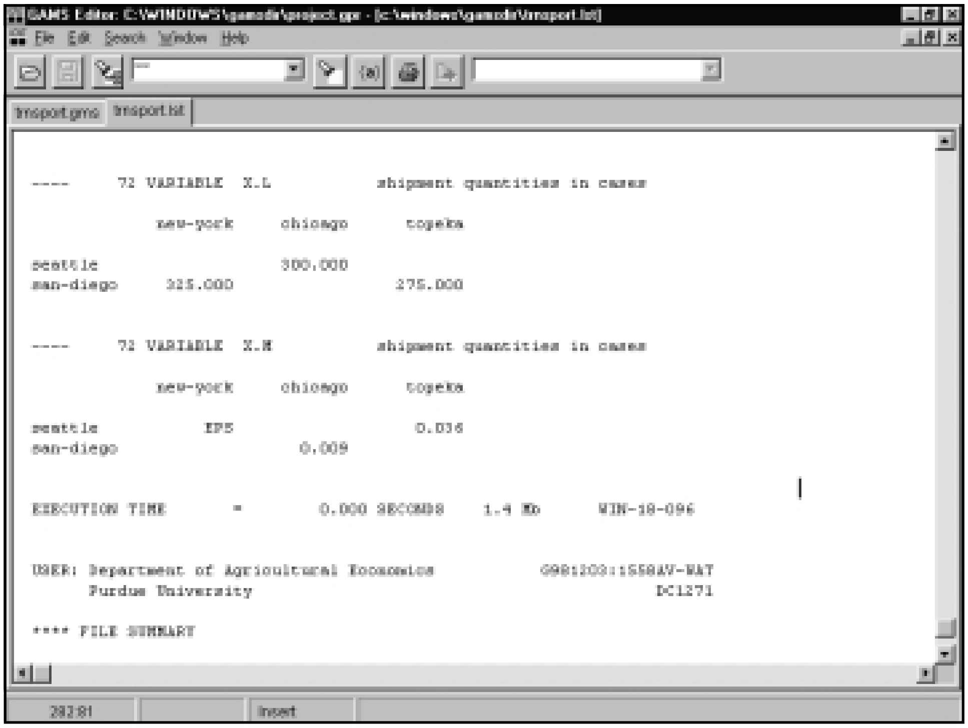
$$\underline{g}_j \le g_j(x) \le \bar{g}_j \qquad j = 1,\ldots,m$$

$$l_i \le x_i \le u_i \qquad i = 1,\ldots,n$$




GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms trnsport.lst
---- EQU DEMAND satisfy demand at market j
LOWER LEVEL UPPER MARGINAL
new-york 325.000 325.000 +INF 0.225
chicago 300.000 300.000 +INF 0.153
topeka 275.000 275.000 +INF 0.126
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 7
A Transportation Problem (TRNSPORT,SEQ=1)
---- VAR X shipment quantities in cases
LOWER LEVEL UPPER MARGINAL
seattle .new-york . . +INF EPS
seattle .chicago . 300.000 +INF .
seattle .topeka . . +INF 0.036
san-diego.new-york . 325.000 +INF .
san-diego.chicago . . +INF 0.009
san-diego.topeka . 275.000 +INF .
LOWER LEVEL UPPER MARGINAL
239:78 Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms trnsport.lst
LOWER LEVEL UPPER MARGINAL
---- VAR Z -INF 153.675 +INF .
Z total transportation costs in thousands of dollars
**** REPORT SUMMARY : 0 NONOPT
0 INFEASIBLE
0 UNBOUNDED
GAMS 2.50A Windows NT/95 12/15/00 14:11:57 PAGE 8
A Transportation Problem (TRNSPORT,SEQ=1)
E x e c u t i o n
---- 72 VARIABLE X.L shipment quantities in cases
new-york chicago topeka
seattle 300.000
san-diego 325.000 275.000
---- 72 VARIABLE X.M shipment quantities in cases
239:78 Insert


GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
---- 72 VARIABLE X.L shipment quantities in cases
new-york chicago topeka
seattle 300.000
san-diego 325.000 275.000
---- 72 VARIABLE X.M shipment quantities in cases
new-york chicago topeka
seattle EPS 0.036
san-diego 0.009
EXECUTION TIME = 0.000 SECONDS 1.4 Mb WIN-18-096
USER: Department of Agricultural Economics G981203:1558AV-WAT
Purdue University DC1271
**** FILE SUMMARY
282:81
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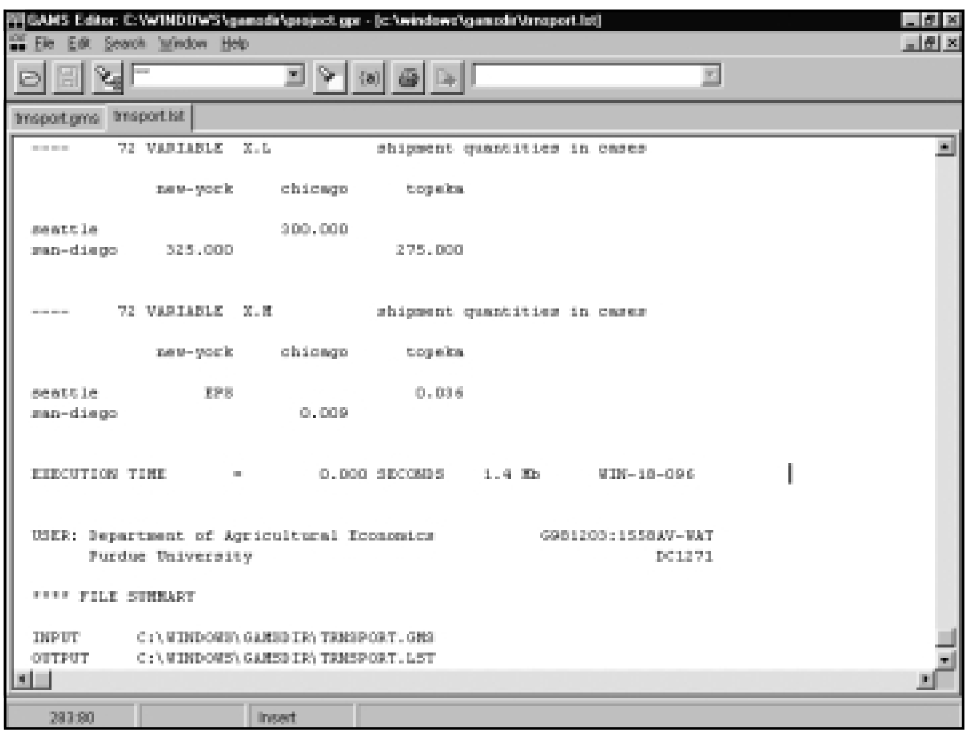
GAMS Editor: C:\WINDOWS\gamsdir\project.gpr - [c:\windows\gamsdir\trnsport.lst]
File Edit Search Window Help
trnsport.gms
trnsport.lst
---- 72 VARIABLE X.L shipment quantities in cases
new-york chicago topeka
seattle 300.000
san-diego 325.000 275.000
---- 72 VARIABLE X.M shipment quantities in cases
new-york chicago topeka
seattle EPS 0.036
san-diego 0.009
EXECUTION TIME = 0.000 SECONDS 1.4 Mb WIN-18-096
USER: Department of Agricultural Economics G981203:1558AV-WAT
Purdue University DC1271
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OUTPUT C:\WINDOWS\GAMSDIR\TRNSPORT.LST
283:80
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