

Another DSP Example

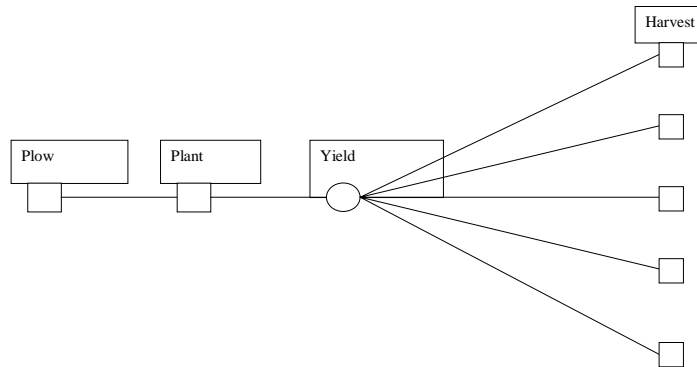
■ Farm Planning with Uncertain Resource Utilization Coefficients

- Revisit the GAMS model similar to PC/LP from Lecture 16
- Modification is to accommodate the dependence of the harvesting rate on the yield, reflecting that the combine must go slower if yields are high than otherwise

PC/LP DSP

- Yields are now stochastic
- Constraint coefficient reflecting combining rate is now stochastic – depends on yield
- Harvesting plan/constraints will have to be flexible to allow efficient operations at different yield levels
- Can we diagram this process?

PC/LP DSP (cont'd.)



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PC/LP DSP (cont'd.)

- Note that decisions (harvest scheduling) are made after the final random event (yields)
- Recall the decision variables from the PC/LP-type model:
 - l $\text{Plow}(\text{Per})$ – Acres to plow in period Per
 - l $\text{Acre}(\text{PPer}, \text{HPer})$ – Acres to plant in period PPer and harvest in period HPer

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PC/LP DSP (cont'd.)

- The random event that we account for is the realization of “ideal yields”
- Ideal yields are either 120, 135, 150, 155, or 190 bu./acre, each with equal probability
- Yield adjustments remain the same (on a percentage basis) regardless of which ideal yield occurs
- This information becomes available *after* plowing and planting have been completed

PC/LP DSP (cont'd.)

- Define an index set to indicate the state of nature
 - | SET State / S1*S5 / ;
- And let the ideal yield parameter be
 - | PARAMETER YLD(STATE) ideal bu. per acre
 - | / S1 120, S2 135, S3 150, S4 155, S5 190 / ;
- The mean of this distribution and the realization for S3 are the same as the base yield for PC/LP model

PC/LP DSP (cont'd.)

- The schedule for plowing must be made independent of the state of nature – $Plow(PPer)$
- The schedule for planting is also made independent of the state of nature
 - | However, the yield adjustment is tied to the combination of planting and harvesting date, and
 - | The base yield shifts with the state of nature

PC/LP DSP (cont'd.)

- To track yield, the plant/harvest scheduling variable must be indexed by planting period, harvesting period, and state
 - | $ACRE(PPer, HPer, State)$
- But, the number of acres planted in any specific planting period must be the same across all states, reflecting that the state is unknown when the planting schedule is determined

PC/LP DSP (cont'd.)

■ To reflect invariance of the planting schedule by date:

- | EQUATIONS
- | PLANT(PPer,State) Planting is equal cross states;
- | PLANT(PPer,State)
- | $\$ \text{Sum}(\text{HPer}, \text{Yield}(\text{PPer}, \text{HPer}, \text{State}))..$
- | $\text{Sum}(\text{HPer} \$ \text{Yield}(\text{PPer}, \text{HPer}, \text{State}),$
- | $\text{ACRE}(\text{PPer}, \text{HPer}, \text{State})) = E =$
- | $\text{Sum}(\text{HPer} \$ \text{Yield}(\text{PPer}, \text{HPer}, \text{State}),$
- | $\text{ACRE}(\text{PPer}, \text{HPer}, 'S1')) ;$

PC/LP DSP (cont'd.)

- | We retain flexibility to shift the harvest schedule as the yield has been effectively realized by the time harvesting begins
- | Let us go back through the PC/LP equations and see how they have changed to incorporate the conditional nature of the problem

PC/LP DSP (cont'd.)

■ The objective becomes expected net return:

| OBJECTIVE..
| NETREV =E=
| Sum(State,1/Card(State)*
| Sum((PPer,HPer)\$Yield(Pper,Hper,State),
| (Yield(PPer,HPer,State)*Price –
| P_HCost)*ACRE(PPer,HPer,State))) ;

PC/LP DSP (cont'd.)

■ Because the *planting* schedule is the same in every state, it is sufficient to constrain land for a single state (e.g., S1)

| LAND..
| Sum((PPer,HPer)
| \$Sum(State,Yield(Pper,Hper,State)),
| ACRE(PPer,HPer,'S1')) =L= LandRhs;

PC/LP DSP (cont'd.)

- The tractor use constraint spans the entire time horizon, including harvest periods

```
| TRACTR(Per,State) ..  
| PlwTrac*Plow(Per) +  
| Sum(HPer$Yield(Per,Hper,State),  
|     PltTrac*ACRE(Per,HPer,State))  
| + Sum(PPer$Yield(PPer,Per,State),  
|     HavTrac*ACRE(PPer,Per,State))  
| =L= Tractor*HRSPERDAYT*DAYS(PER) ;
```

PC/LP DSP (cont'd.)

- As with the Land constraint, it is sufficient to impose the sequencing constraint for plowing and planting for only one state

```
| SEQ(Per) ..  
| Sum(Period$(ord(Period) le ord(Per)),  
|     PLOW(Period)) =G=  
| Sum(Pper$(ord(Pper) le ord(Per)),  
|     Sum(Hper$yield(Pper,Hper,'S1'),  
|     ACRE(Pper,Hper,'S1')));
```

PC/LP DSP (cont'd.)

- The big difference between this model and the PC/LP model is that the combine working rate depends on yield – Here, the combine working rate is specified in hours/bu

- SCALAR
- COMB Combine working rate (hrs per bu)
- / 0.0022222 / ;

- To get total hours used, we multiply (hrs/bu)* (bu/acre)* (acre) to get (hrs)

PC/LP DSP (cont'd.)

- The combine use constraint becomes:

- HARVEST(HPer,State)\$
- Sum(PPer, Yield(PPer,HPer,State)) ..
- Sum(PPer\$Yield(PPer,HPer,State),
- Comb*Yield(PPer,HPer,State)*
- ACRE(PPer,HPer,State))
- =L= CombRhs(HPer) ;

- Notice that the coefficients on ACRE() vary by State and that the constraint is indexed by State

PC/LP DSP (cont'd.)

- Compare the solution for the plowing schedule between the PC/LP model and PC/LP DSP

- | PC/LP

- | ---- 148 VARIABLE PLOW.L

- | MARAPR 699.36, APR3 50.64

- | PC/LP DSP

- | ---- 175 VARIABLE PLOW.L

- | MARAPR 699.36, APR3 50.64

PC/LP DSP (cont'd.)

- Plant/Harvest Schedule

- | ---- 176 VARIABLE ACRE.L Acreage

- | S1 S2 S3 S4 S5

- | APR3.SEP4 74.21 74.21 74.21 74.21 74.21

- | APR4.OCT1 264.45 235.07 211.56 204.74 167.02

- | APR4.OCT2 14.32

- | APR4.NOV1 5.49 49.19 72.69 79.52 117.23

- | MAY1.OCT2 284.26 253.98 218.29 207.93 150.67

- | MAY1.NOV1 30.28 65.96 76.33 133.58

- | MAY2.OCT2 107.27 107.27 107.27 107.27 107.27

PC/LP DSP (cont'd.)

- Notice that if we sum across harvest dates, the ACRE variables have the following sub-totals by plant date:

	S1	S2	S3	S4	S5
APR3	74.21	74.21	74.21	74.21	74.21
APR4	284.26	284.26	284.26	284.26	284.26
MAY1	284.26	284.26	284.26	284.26	284.26
MAY2	107.27	107.27	107.27	107.27	107.27

- Why are these all the same across states?
- How do they compare to the PC/LP results? (Same!)

PC/LP DSP (cont'd.)

- Now, what if we sum ACRE across planting dates?

	S1	S2	S3	S4	S5
SEP4	74.21	74.21	74.21	74.21	74.21
OCT1	264.45	235.07	211.56	204.74	167.02
OCT2	405.85	361.25	325.57	315.2	257.95
NOV1	5.49	79.46	138.66	155.84	250.82

- How do these compare to the PC/LP results?
- (74.21, 211.56, 314.77, 149.46)
- Notice the differences in Oct2 and Nov1

PC/LP DSP (cont'd.)

- Now consider the shadow prices/penalty costs/
Lagrange multipliers and their interpretation
- For the Land constraint, the marginal value of an
additional acre is:
 - ▮ \$180.65 for the PC/LP model
 - ▮ \$182.51 for the PC/LP DSP model
- This is the value of having one more acre of land for
the current year only (i.e., the value of a rented acre)

PC/LP DSP (cont'd.)

- Now consider the shadow prices on combine
capacity (HARVEST.M):
 - ▮ OCT1 \$72.97, OCT2 \$52.88 for PC/LP
 - ▮ OCT1.S1 \$14.59, OCT2.S1 \$10.79 for PC/LP
DSP
- Q: Why so much lower for PC/LP DSP?
- A: This is the conditional value – increasing the RHS
of the constraint only increases it one fifth of the time

PC/LP DSP (cont'd.)

- Notice that
 - | $\$14.59 \times 5 = \72.95 and
 - | $\$10.79 \times 5 = \53.95
- These are pretty close to the values of \$72.97 and \$52.88 that we observed for the PC/LP model
- In general, if we want the rental value of a “conditional factor” we should divide by the probability of the condition occurring

DSP – Summary

- Discrete Stochastic Programming
 - | Allows us to incorporate uncertainties not only in our objective, but also in our constraints (Both RHS and coefficients)
 - | Reactions to realizations of random variables can be incorporated through the introduction of conditional variables and constraints

DSP – Summary

- | The main focus should be on the initial decisions and resource values (the unconditional variables and constraints associated with the leading decision node)
- | When determining what one should do at decision node subsequent to the first, one should really build a new DSP model that looks forward in time from that decision node