

DSP Example

- Again, we illustrate the formulation process with an example -- a portfolio problem with revisions of the portfolio allocations

I Problem features:

- The decision maker starts with \$1000
- The planning horizon is one year
- There are two decision points, now and six months from now

DSP Example (cont'd.)

- There are three investment options under consideration: a stock mutual fund (S), a bond mutual fund (B), and cash (C)
- There are no transactions costs
- The decision maker is an expected utility maximizer with constant relative risk aversion equal to 2
- At the end of the first six months, one of four states of nature occurs

DSP Example (cont'd.)

Returns by State of Nature

State	Prob.	S	B	C
1	0.20	1.15	1.05	1.00
2	0.40	0.95	1.03	1.00
3	0.30	1.10	1.04	1.00
4	0.10	1.04	1.07	1.00

- Note that state 1 is good for stocks, state 2 is bad for stocks, state 3 is intermediate for both stocks and bonds, and state 4 is good for bonds and mediocre for stocks
- Note that cash is a sure thing

DSP Example (cont'd.)

- I For convenience, these same 6-month returns states are used for the second 6-months, but the probabilities are different and depend on the outcomes in the first 6-months

Conditional Probabilities of 2nd 6M. States

State (1 st 6M.)	State (2 nd 6M.)	1	2	3	4
1	1	0.1	0.5	0.2	0.2
2	1	0.3	0.2	0.4	0.1
3	1	0.2	0.3	0.3	0.2
4	1	0.3	0.3	0.3	0.1

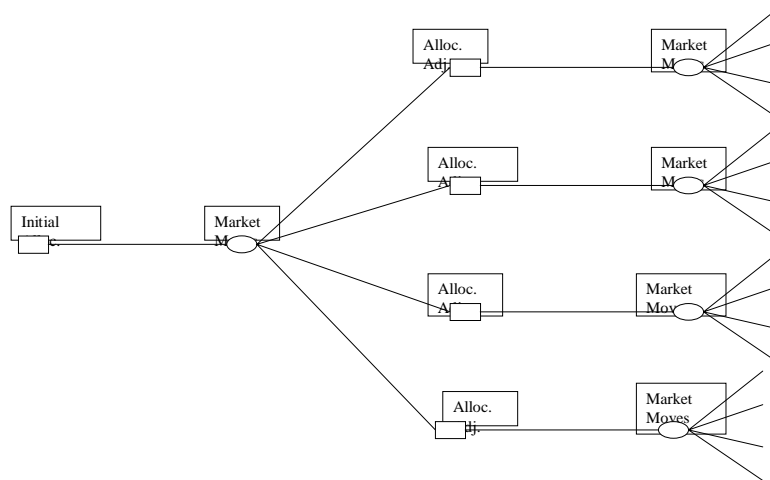
DSP Example (cont'd.)

- Can we diagram this problem?
- What are the initial decisions?
- What are the initial constraints?
- What are the first random outcomes to occur?
- What decisions are made in the second stage?
- What are the constraints limit choices in the second stage?

DSP Example (cont'd.)

- What random events occur after the second stage decisions?
- How many states of nature are there for wealth at the end of the year?
- How can we keep track of the fact that second stage decisions are conditional on the outcomes in the first stage?
- How can we keep track of the fact that second stage constraints
 - Are conditional, and
 - Only limit the variables that are likewise conditional?

DSP Example (cont'd.)



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DSP Example (cont'd.)

■ Formulation

■ What kinds of sets might we need?

- | SETS
- | i States of nature / 1*4 /
- | v Investment vehicles / s,b,c /
- | cash(v) Just cash / c /
- | notc(v) Other than cash / s,b / ;

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DSP Example (cont'd.)

■ What are the initial (unconditional decisions)?

| POSITIVE VARIABLES

| $x(v)$ Initial investment allocation ;

■ What are the initial constraints?

| EQUATIONS

| limcap1 Initial capital limitation ;

| limcap1 .. $\text{sum}(v, x(v)) = 1000$;

DSP Example (cont'd.)

■ What are the first random outcomes to occur?

| TABLE r(i,*) Returns for first 6 months

| prob s b c

| 1 .2 1.15 1.05 1

| 2 .4 0.95 1.03 1

| 3 .3 1.10 1.04 1

| 4 .1 1.04 1.07 1 ;

DSP Example (cont'd.)

I What decisions are made in the second stage?

- I POSITIVE VARIABLES
- I $s(v,i)$ Sales at mid-year
- I $b(v,i)$ Purchases at mid-year
- I $y(i)$ Cash held for second 6 mos. ;

I What constrains those decisions?

- I EQUATIONS
- I $\text{limsell}(i,v)$ Limits on selling at mid-year
- I $\text{limcap2}(i)$ Mid-year capital limitations

DSP Example (cont'd.)

- I $\text{limsell}(i,\text{notc}) \dots s(\text{notc},i) \leq x(\text{notc})$;
- I $\text{limcap2}(i) \dots \text{sum}(\text{notc},b(\text{notc},i)) + y(i) \leq$
I $\text{sum}(\text{cash},x(\text{cash})) + \text{sum}(\text{notc},r(i,\text{notc}) * s(\text{notc},i))$;

I What random events occur after the second stage decisions?

DSP Example (cont'd.)

| TABLE q(i,j,*) Conditional returns distribution for
| * second 6 months
| 1.prob 2.prob 3.prob 4.prob
| 1 .1 .5 .2 .2
| 2 .3 .2 .4 .1
| 3 .2 .3 .3 .2
| 4 .3 .3 .3 .1 ;
| $q(i,j,v) = r(j,v)$;

- | (Note the conditional nature of this distribution)

DSP Example (cont'd.)

- | How many states of nature are there at the end of the year? ($4 \times 4 = 16$)
- | How do we keep track of the fact that second stage decisions are conditional on the outcomes in the first stage? (By indexing them by the first stage state)
- | How can we keep track of the fact that second stage constraints
 - | Are conditional?
 - | Only limit the variables with the same conditions?
 - | (Again, indexing.)

DSP Example (cont'd.)

- For accounting purposes, these variables and constraints are handy:

- POSITIVE VARIABLES

- $w(i,j)$ Wealth at year's end

- VARIABLES

- eu Objective ;

- EQUATIONS

- $wdef(i,j)$ Wealth definition constraints

- obj Expected utility definition constraint ;

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DSP Example (cont'd.)

- $wdef(i,j) .. w(i,j) = e= \sum(notc, q(i,j, notc)) * (r(i, notc) * (x(notc) - s(notc, i)) + b(notc, i)) + y(i) ;$

- $obj .. eu = e= 1000 * \sum((i,j), q(i,j, 'prob')) * r(i, 'prob') * (\log(w(i,j))) \$(\rho eq 1 + (w(i,j))^{(1-\rho)/(1-\rho)} \$(\rho ne 1)) ;$

- Note the use of $w(i,j)$ so we can be sure wealth stays positive

- Note that the special case $\rho=1$ is handled correctly

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DSP Example (cont'd.)

- Do the dynamics of these decisions matter?
- Does the conditional nature of response matter?

■ Experiments

- Compare the results of the DSP to the case where no mid-year adjustment is permitted
- Compare the results of the DSP to the case where mid-year adjustment is allowed but must be the same independent of what happens in the first six months

DSP Example (cont'd.)

■ Implementing experiment #1

- $s.fx(v,i) = 0$;
- $b.fx(v,i) = 0$;
- Note that we are not allowing purchases or sales at mid-year

DSP Example (cont'd.)

I Implementing experiment #2

I equations

I $\text{unifb}(v,i)$ Equalize second period purchases

I $\text{unifs}(v,i)$ Equalize second period sales ;

I $\text{unifb}(v,i) \dots b(v,i) = e = b(v, '1')$;

I $\text{unifs}(v,i) \dots s(v,i) = e = s(v, '1')$;

I Note purchases and sales must all equal the purchase/sale that occurs after state 1 is realized

DSP Example (cont'd.)

I Results – Initial allocations

	Stocks	Bonds
I Expt. #1	902.387	97.613
I Expt. #2	363.645	636.355
I DSP	252.216	747.784

I Note that the effect on the initial allocations is quite large – Do you expect conditional transactions to be significant?

DSP Example (cont'd.)

I Results – conditional purchases and sales

- I Expt. #1 – No purchases or sales allowed
- I Expt. #2
 - I Bond sales at mid-year 636.355 shares
 - I Stock purchases at mid-year 655.445 shares
 - I Cash holdings by state of nature 12.727, 0, 6.364, and 25.454 for state 1, 2, 3, and 4

DSP Example (cont'd.)

I DSP conditional purchases and sales

I State	1	2	3	4
I Sell Stock	252.2			
I Sell Bond		747.8	630.2	747.8
I Buy Stock		770.2	655.4	800.1
I Buy Bond	290.0			

DSP Example (cont'd.)

- Expected utility levels

- | Expt. #1: -0.915

- | Expt. #2: -0.913

- | DSP: -0.908

- What do these mean?

DSP Example (cont'd.)

- Alternative concept – certainty equivalent

- | Define: the certainty equivalent is the amount of certain money that makes the decision maker indifferent between the distribution of returns and the certain amount

- | Mathematically,

- | $CE = U^{-1}\{E[U(W)]\}$

DSP Example (cont'd.)

■ Certainty Equivalents (\$):

■ Expt. #1 1092.855

■ Expt. #2 1095.138

■ DSP 1101.565