Single-Agent, Static, Deterministic Models

- Producers -- Neoclassical theory views producers as maximizers of profits subject to (at least) technological restrictions

$$\underset{y,x}{\text{maximize}}\ p'y - w'x$$
$$\text{subject to: } (y,x) \in \Omega$$

where *y* is a vector of outputs, *x* a vector of inputs, *p* a vector of output prices, *w* a vector of input prices, and Ω defines technology.

Producer Models

- This problem is trivial except for Ω.
- What has been assumed about competition/ firm size in this model?
  - How can you tell?
  - Can you relax this assumption? If so, how does the model change?

Describing Technology

- Description of technology (Ω)

  - The simplest case is when there is a single output -- i.e., the vector *y* has just one component.

$$\underset{x \geq 0}{\text{maximize}} \quad pf(x) - w'x$$

  where $y = f(x)$ is a scalar function of input use x.

Single-output Technology (cont'd.)

- Note that the function $f(x)$ describes the *efficient frontier* for the production system.
  - What does *efficient* mean here?
- Given the above problem, we would probably like for $f(x)$ to be strictly increasing in each of its arguments and concave.
  - What kind of functions are like this?

Single-output Technology (cont'd.)

- Can we use the functions we just reviewed as utility functions for production relationships?
- Do the modeling properties of these functions change?
  - (As you think about this, change the words "consumer prices" to "input prices". Also think about what happens if output price is "too low".)

Single-output Technology (cont'd.)

- Do the benchmarking procedures for *f*(*x*) change?
  - (Hint: the answer is "yes". Why?)
- Consider the CES function as a production function. Recall that this function is generally written as:

$$y = f(x_1, \cdots, x_n) = \alpha \left(\sum^{n} \beta_i x_i^{-\rho} \right)^{-1/\rho}$$

Single-output Technology (cont'd.)

- In the case where we were benchmarking a utility function, α did not matter.
  - Does it matter now?
  - How do you think we should set α ?
  - How does this look for the CES?

Single-output Technology (cont'd.)

$$\alpha = y\left(\sum^{n} \beta_i x_i^{-\rho}\right)^{1/\rho}$$

- where y is benchmark output, and the x_i are benchmark inputs.
- For other functions (e.g., Cobb-Douglas), benchmarking production is modified in a similar way.

Single-output Technology (cont'd.)

- Returns to Scale
  - Recall from theory that constant returns to scale implies zero profits for production processes that are employed --
    - Positive profits suggests that production should expand, and
    - Negative profits suggests that production should shut down

Single-output Technology (cont'd.)

- When the data is inconsistent with zero profits, it will usually be because profits are positive.
  - In this case, we typically define an additional input (e.g., "capital") that exactly accounts for profits.
    - Units for this type of input are typically undefined -- only the value (price times quantity) of the input can be assessed
    - Including the additional input, technology has zero profits and is consistent with constant returns to scale

Single-output Technology (cont'd.)

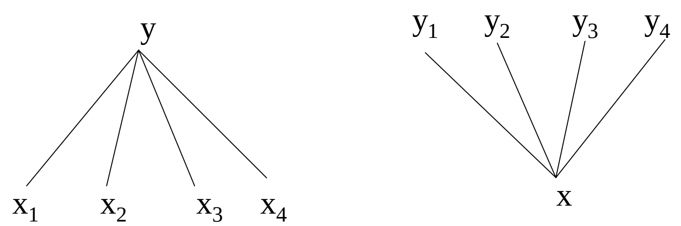
- When profits are negative, we must have gotten something wrong -- our accounting is incomplete
  - How do you explain why farmers continued to raise hogs in December 1999 when prices were not even high enough to pay for feed?
  - How do you explain why some .com's remain in business in spite of never having made a profit?

Single-output Technology (cont'd.)

- The types of things that can go wrong when profits appear to be negative --
  - Double counting of input costs for inputs that cannot be allocated
  - Failure to account for joint outputs (either static or dynamic)

Single Input/Multiple Output Technology

- Imagine turning a single output/multiple input technology upside down, and you have a single input/multiple outputs (think of the tree)



One Input/Multiple Outputs (cont'd.)

- The profit maximizing firm's problem then becomes:

$$\underset{(y,x)\in\Omega}{\text{maximize}} \;\; p'y - wx$$

- Note that *y* is being treated as a vector in this formulation, while *x* is being treated as a scalar (observe the transpose)

One Input/Multiple Outputs (cont'd.)

- If the frontier of the efficient input level for a given output combination can be specified as a functional relationship, then this problem can be stated as:

$$\underset{y}{\text{maximize}} \;\; p'y - wf(y_1, y_2, \ldots, y_m)$$

- What properties do we want *f*() to satisfy?

One Input/Multiple Outputs (cont'd.)

- The most commonly used nonlinear specification for *f*() is the CET.
  - Recall that the CET looks identical to the CES, but that $\rho < -1$
  - In this case, *f*() is
    - Strictly increasing in each output and
    - Convex

One Input/Multiple Outputs (cont'd.)

- Benchmarking the CET proceeds much the same as for the CES
  - An elasticity of transformation (ϵ) is typically obtained from the literature or assumed
  - The value of the exponent is computed as:

$$\rho = (1-\varepsilon)/\varepsilon$$

One Input/Multiple Outputs (cont'd.)

- Then the β_i are computed via the relationships

$$\beta_i = \bar{p}_i \left(\bar{y}_i \right)^{1/\varepsilon} \left[\sum^{m} \bar{p}_j \left(\bar{y}_j \right)^{1/\varepsilon} \right]^{-1}$$

One Input/Multiple Outputs (cont'd.)

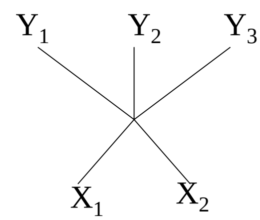
- As with the CES case for one output, the constant term α must be set

$$\alpha = x\left[\sum_{i=1}^{m} \beta_i y_i^{-\rho}\right]^{1/\rho}$$

Multiple Inputs *and* Outputs

- One way to reflect multiple inputs and outputs is by combining CES and CET specifications
  - The CES describes how to get from multiple inputs to a single, aggregate input, and
  - The CET describes how to get from the single, aggregate input to multiple outputs
    - What does the tree look like?

Multiple Inputs and Outputs (cont'd.)



- What is input/output separability?
- How would you benchmark this system? (Hint: think about what we do with nested functional forms.)

Simpler&More Complex -- Linear Prod.

- Linear production systems can deal with *all* of the above cases
  - Multiple input/single output (most common),
  - Single input/multiple output (least common), and
  - Multiple input/multiple output

Linear Production (cont'd.)

- To begin, consider the single output case
- Recall that one way to write the Leontief specification of technology (output as a function of input levels) is via the optimization problem:

$$\underset{y}{\text{maximize}}\ y$$

$$\text{subject to: } a_i y \le x_i \quad i = 1, \ldots, n$$

Linear Production (cont'd.)

- Linear technology specification virtually *never* appears in a model directly in this form
  - Note that the above is a pure technology specification
  - In an economic model, the technology specification will be embedded within a model of agent behavior

Linear Production (cont'd.)

- For example, this technology specification might be embedded in a model of producer behavior based on profit maximization --

$$\underset{y, x_1, x_2, \ldots x_n}{\text{maximize}} \; py - \sum_{i=1}^{n} w_i x_i$$

$$\text{subject to: } a_i y \leq x_i, \; y \geq 0, \; x_i \geq 0, \; i = 1, \ldots, n$$

  - This is not a very interesting formulation either. What is wrong with it?

Linear Production (cont'd.)

- It is not very interesting because:
  - For a given level of *y*, input use is defined ($x_i = a_i y$)
  - The solution can be characterized by three cases:
    - Zero profits -- level of *y* is indeterminate
    - Positive profits -- no finite value of *y* is optimal
    - Negative profits -- $y=0$ is optimal

Linear Production (cont'd.)

- Few production systems are infinitely scalable at fixed prices
  - The labor base may be fixed
  - The capital base may be fixed
  - Management capacity may be fixed
  - This list is far from exhaustive

Linear Production (cont'd.)

- In the presence of fixed inputs, the problem becomes:

$$\underset{y, x_1, x_2, \ldots x_t}{\text{maximize}} \quad py - \sum_{i=1}^{n} w_i x_i$$

- In this modified formulation, the x_i, $i=t+1,\ldots,n$ are no longer variables -- they are the fixed levels for inputs

Linear Production (cont'd.)

- If there are several fixed factors, then one will limit the level of *y* (assuming profitability) -- i.e., one gives the minimum value for $y=x_i/a_i$

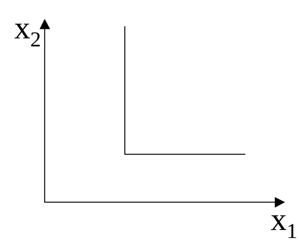
- The shadow price for the fixed input will equalize the level of profit to the value of the input (*j*):

$$py - \sum_{i=1}^{t} w_i x_i = \pi_j x_j$$

Linear Production (cont'd.)

- What happens when there is a tie? (What is this phenomenon called?)

- This model is still pretty useless as there is no substitution with a Leontief production system



- How should we fix it?

Linear Production (cont'd.)

$$\underset{y_1,\ldots,y_m,x_1,\ldots x_t}{\text{maximize}} \quad p\sum_{j=1}^{m} y_j - \sum_{i=1}^{n} w_i x_i$$

$$\text{subject to:} \sum_{j=1}^{m} a_{ij} y_j \le x_i,\; y_j \ge 0,$$

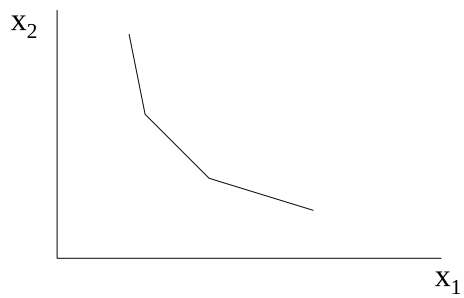
$$x_i \ge 0,\; i = 1,\ldots,n$$

Linear Production (cont'd.)

- This formulation has one output and a mixture of fixed and variable inputs
- What do the isoquants look like?

Linear Production (cont'd.)

- Isoquants with two inputs, linear production and multiple technologies:



- What happens to the allocation of net profits to fixed inputs?

Linear Production (cont'd.)

- It is a small step to take this formulation to multiple outputs:

$$\underset{z_1,\ldots,z_q,y_1,\ldots,y_m,x_1,\ldots x_t}{\text{maximize}} \sum_{k=1}^{q} p_k z_k - \sum_{i=1}^{t} w_i x_i$$

$$\text{subject to: } \sum_{j=1}^{m} a_{ij} y_j \leq x_i, \; i = 1,\ldots,n$$

$$\sum_{j=1}^{m} b_{jk} y_j \geq z_k, \; k = 1,\ldots,q$$