Agricultural Economics 652

Application of Quantitative Analysis: Mathematical Programming

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http://www.agecon.purdue.edu/academic/agec652/index.htm

Ag. Econ. 652 Prerequisites

- Agricultural Economics 552, or
- A thorough understanding of matrix algebra, differential calculus, and linear programming
- See the instructor if your background does not satisfy the prerequisite

Objectives

- Theory and practical aspects of the formulation and solution of optimization models
  - Optimality conditions
  - Convex analysis
  - Modeling approaches

Objectives (cont'd.)

- Modeling
  - Agents/Systems
    - Consumers
    - Producers
    - Sector
  - Features
    - Risk
    - Dynamics
    - Policy/Contracts

Hands-On

- Homework *requires* use of GAMS modeling language
- Hands-on experience at the formulation, solution and interpretation of optimization models
- GAMS usage addressed in the course

Book – Reference Only

- Course will be taught primarily on the basis of these notes
- GAMS User's Guide purchase optional
- A better solution: access through
- http://www.gams.com/docs/pdffiles.htm

Added Reading

- Convex Analysis/Optimality Conditions: P.E. Gill, W. Murray, and M. Wright, Practical Optimization, 1981
- Journal articles will be assigned as appropriate during the semester

Exams/Grades

- Midterm examination (15%)
- Final examination (30%)
- Homework and quizzes (30%)
- Class project (20%)
- Class participation (5%)

Homework Policy

- "On Time" means turned in before grading begins (never before 5:00pm on due date)
- Papers that are not on time
  - Will be docked 10% if turned in before solutions have been distributed
  - Will be marked, but not credit given if turned in after solutions have been distributed

Homework Policy (cont'd.)

- Discussing together is permitted
- Write programs and interpretations independently
  - Academic dishonesty is dealt with harshly at Purdue – see the Website following for further information: http://www.purdue.edu/ODOS/bacinteg.htm

Computer Usage

- Mathematical Programming is pervasive in Applied Economics
  - Some models are analytical and allow derivation of unambiguous qualitative impacts
  - Many models are not amenable to analytics – hence, numerical approaches are needed

Computer Usage (cont'd.)

- The computer is a fundamental tool for numerical analysis
- Each student is required to learn and use the GAMS modeling language
- GAMS is available over the Department's network, in the KCIC computer labs on the seventh floor, and for free download at http://www.gams.de/5download/cd.htm

Students with Disabilities

If you have a disability which requires some special accommodation, please make an appointment within the first three weeks of the semester for a visit to my office to discuss the appropriateness of the instructional methods in this class, or any academic adjustments you may need. I have found it possible to make adjustments in the past, but it is important that we talk about this at the beginning of the semester.

Agricultural Economics 652

- What is Mathematical Programming?
- The old view —
- "The ability to state general objectives and then find optimal policy solutions to practical decision problems of great complexity..." (Dantzig, 1982).

(What is Math Programming?)

- A different perspective —
- Mathematical Programming is an approach to analysis based on optimization. This approach involves one or more of the following:

(What is Math Programming?)

- Formulation of real world phenomena within a mathematical model,
- Analysis of the theory behind a mathematical model,
- Analytical or numerical solution of a model, and
- Interpretation and evaluation of model results.

(What is Math Programming?)

- **Optimization**
  - Determining the minimum (or maximum) of a mathematical function (the objective) subject to a set of restrictions (the constraints).
  - Variables may be discrete or continuous.

(What is Math Programming?)

- **Other problems**
  - Systems of equations
  - Complementarity problems

Optimization Problems

- Mathematically, the problem is usually stated as "find a vector *x*" that solves

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{subject to}: \; g_i(x) \geq 0 \quad i = 1,...,m \end{aligned}$$

- where $f(x)$ (the objective) and $g_i(x)$ (the constraints) are continuous, differentiable functions of the arguments x.

Special Cases

- **Linear Programming**
  - The objective is linear and
  - The constraint functions are affine (linear plus a constant)
- **Nonlinear Programming**
  - More general case where the objective and constraint functions may be linear or not
  - For practical work, a number of restrictions are usually imposed on these problems:

(Nonlinear Programming Cont'd.)

- The list of variables and constraints is finite (i.e., no infinite horizon or continuous time problems)
- Continuity (and usually differentiability) of the objective and constraint functions
- Local optima are of sufficient interest (numerical methods only find local optima; analysis can at times guarantee global optimality)

Integer/Mixed Integer Linear Prog.

- Restriction of all (the **Integer** case) or some (the **Mixed Integer** case) variables to be whole numbers
  - **Binary** case variables can only take on the values zero or one
  - Effort required to compute precise optima can be extremely large (approximate solutions often suffice)

Math Prog. in Applied Econ.

- Used to discover physical and behavioral relationships, or
- To determine the consequences of changes in the physical, economic, or regulatory environment on an economic system.

Use #1

- Isn't this econometrics?!?
- What problems are of interest in econometrics?
  - *Least* squares
  - *Maximum* likelihood
  - *Maximum* entropy

Use #2

- Micro- and Macro-economics motivate the behavior of individuals as optimizers
  - Consumer theory – utility maximization
  - Producer theory – profit maximization
  - Sector/economy – “efficiency”

Recent Applications of Math. Prog.

- Estimation or analysis of new or tough econometric problems
  - Maximum likelihood – restricted domain for ML objective
  - Maximum entropy – analysis of relation to least squares
  - New functional forms/models – motivation of consumer problems with more than just a budget constraint

(Recent Applications)

- Evaluation of the impacts of change based on neoclassical agent behavior
  - Impact of technology on nutrition and marketed surplus
  - Impact of contract structure on input use on crops
  - Valuation of technology for insect control

Math. Prog. Analysis and Practice

- Analysis
  - Helps us to understand the properties of our models and
  - Gives us insight into the properties of their solutions
- Practical Knowledge
  - Standard techniques for formulation of phenomena
  - Appropriate implementation strategies