Dynamic, Deterministic Models

- Key to Correct Formulation of Dynamic Models: UNITS!
  - In addition to bushels, tons and kilograms, units now include when (i.e., a date)
    - Bushels sold in December
    - Kilograms of fertilizer applied in the first week of the rainy season
    - Warehouse capacity available today

Why is dynamic different?

- The other distinguishing feature of dynamic models is the one-way flow of commodities and value
  - Physical commodities flow forward through time – inventory storage
  - Value (shadow prices) flow backward through time (i.e., today's value of a stock of a commodity or resource depends not only on value for use today but also on value for use in the future)

Types of Dynamic Models

- The two principle types of dynamic models are
  - Scheduling models -- focus on how to allocate fixed resources across time
  - Inventory models -- focus on how to manage flows of resources across time

Scheduling Models

- With scheduling models, we typically have a number of fixed resources whose use must be *scheduled* over time
  - Land use (crop mix) may be scheduled over one or more crop years
  - Machinery use may be scheduled over planting and/or harvesting seasons
  - Utilization of livestock facilities may be scheduled

Scheduling Models (cont'd.)

- Warehouse/grain bin utilization may be scheduled over time
- Personnel usage (task assignment) may be scheduled
- Transportation facilities use may get scheduled (e.g., truck, trains, planes, depots, gates)
- Maintenance may get scheduled

Inventory Models

- Inventory Models -- The other big type of dynamic model
  - Focus is on managing flows of “commodities” over time
  - These commodities are storable over time (perhaps within limits due to perishability)
  - As with most flow systems, leakage (storage losses) may be a problem

Inventory Models (cont'd.)

- Examples of inventory models include –
  - Resource (renewable or nonrenewable) management (oil and other fossil fuels, soil, water, fish, etc.)
  - Storage of commodities (timing/locations of supply, and timing/locations of demand in combination with differences in storage costs/quality create opportunities)
  - Inputs to manufacturing processes must be available in a timely fashion to avoid production shortfalls and/or idling of productive resources

Inventory Example – A Grain Marketing System

- Consider the example of the Zambian maize marketing system (Mwanaumo, 1994)
- System features:
  - Maize is produced at different levels of intensity in different parts of the country
  - Maize is consumed at different rates in different parts of the country (different from prod. pattern)

Maize Marketing System (cont'd.)

- The government controls transport and marketing of maize around the country
- There are inter-regional shipping losses independent of source and destination
- The regional network is not directly, fully connected – shipments from Kabwe to Solwezi may not be possible, but shipment from Kabwe to Kitwe and then Kitwe to Solwezi are feasible

Maize Marketing System (cont'd.)

- Supply becomes available primarily in the second quarter of the marketing year with some additional supplies in the first and third quarters
- The maize supplied to the marketing system by farmers must be processed before delivery to consumers
- There are two processed products – breakfast and roller

Maize Marketing System (cont'd.)

- There are capacity restrictions in each region for storage and processing
- Storage costs vary by region
- Transportation costs between regions are linear in distance between the regions

Maize Marketing System (cont'd.)

- This is an ideal case for an inventory model
  - Multiple locations
  - Multiple products
  - Capacity limits
- The goal will be to minimize the sum of
  - Transportation costs
  - Storage costs
  - Processing costs
  - Subject to satisfying consumer demand

Maize Marketing System (cont'd.)

- Consider the region I, period T maize commodity balance constraint:
- MAIZE(I,T) .. SUPPLY(I,T)
-     + STOREM(I,T-1) *(1-SLOSS)
-     + SUM(J,SHIPM(J,I,T))*(1-TLOSS)
-     =G= SUM(J,SHIPM(I,J,T)) + STOREM(I,T)
-     + 1/YIELDR*MAKER(I,T) + 1/YIELDB*MAKEB(I,T)
- This is a standard "sources at least as great as uses" constraint

Maize Marketing System (cont'd.)

- The units of this constraint are metric tons of maize in region I, period (quarter) T
- SHIPM(I,J,T) denotes shipments of maize from I to J in period T
- STOREM(I,T) denotes storage of maize in region I from period T to period T+1
- MAKER(I,T) denotes the number of metric tons of roller produced in region I, period T

Maize Marketing System (cont'd.)

- YIELDR is the yield of roller (metric tons of roller produced per metric ton of maize)
- MAKEB(I,T) denotes the number of metric tons of breakfast produced in region I, period T
- YIELDB is the yield of breakfast (metric tons of breakfast produced per metric ton of maize)
- SUPPLY(I,T) denotes the metric tons of maize coming into the system in region I, period T

Maize Marketing System (cont'd.)

- TLOSS denotes the shipping loss factor – for each ton of maize shipped from the source, (1-TLOSS) tons of maize arrive at the destination
- SLOSS denotes the storage loss factor – for each ton of maize stored in period T, (1-SLOSS) tons of maize are available in period T+1

Maize Marketing System (cont'd.)

- Now consider the contributions of *these variables* to the objective:
- SUM((I,J,T),TCOSTM(I,J)*SHIPM(I,J,T)
-   + SUM((I,T),SCOSTM(I,T)*STOREM(I,T)
-   + SUM((I,T),MCOSTR(I)*MAKER(I,T))
-   + SUM((I,T),MCOSTB(I)*MAKEB(I,T))
- These are the costs for shipping maize, storing maize, processing roller and breakfast

Maize Marketing System (cont'd.)

- As we have seen before, the transportation variables (SHIPM(I,J,T)) link the value of maize across regions
- The FOC for SHIPM(I,J,T) is:
- TCOSTM(I,J) + MAIZE.M(I,T)
-   – MAIZE.M(J,T)*(1 – TLOSS) $\geq$ 0

Maize Marketing System (cont'd.)

- or
- MAIZE.M(I,T) $\geq$ MAIZE.M(J,T)*(1 – TLOSS)
- – TCOSTM(I,J)
- Note that this relationship will be an equality if SHIPM(I,J,T)>0 and the inequality may be strict otherwise
- This is an arbitrage condition that relates value of maize across regions

Maize Marketing System (cont'd.)

- The storage activities have a similar function, but they relate values across time periods
- Recall that storage is limited by capacity:
- STOREM.UP(I,T) = SCAPM(I)
- The FOC for STOREM(I,T) is
- SCOSTM(I) + MAIZE.M(I,T)
- – MAIZE.M(I,T+1) *(1 – SLOSS)
- – STOREM.M(I,T) $\geq$ 0

Maize Marketing System (cont'd.)

- or
- MAIZE.M(I,T) $\geq$ SCOSTM(I)
-     + MAIZE.M(I,T+1)*(1 – SLOSS) – STOREM.M(I,T)
- This will be an equality if 0<STOREM(I,T), but left- and right-hand sides may not be equal otherwise
- This too is an arbitrage condition, but now it is across time periods within a region (Notice STOREM.M(I,T)<0)

Maize Marketing System (cont'd.)

- To see the other relationships, we introduce the balance equations for processed products
- For "roller"
- ROLLER(I,T) .. MAKER(I,T)
-    + STORER(I,T-1) *(1 – SLOSS)
-    + SUM(J,SHIPR(J,I,T))*(1 – TLOSS)
-    =G= SUM(J,SHIPR(I,J,T)) + STORER(I,T)
-          + DEMANDR(I,T)

Maize Marketing System (cont'd.)

- and for "breakfast"

- BFAST(I,T) .. MAKEB(I,T)
-     + STOREB(I,T-1) *(1 – SLOSS)
-     + SUM(J,SHIPB(J,I,T))*(1 – TLOSS)
-     =G= SUM(J,SHIPB(I,J,T))
-         + STOREB(I,T) + DEMANDB(I,T)

- Again, these are "sources at least as great as uses" constraints

Maize Marketing System (cont'd.)

- The objective contributions for the new variables are:

- SUM((I,J,T),TCOSTR(I,J)*SHIPR(I,J,T))
-    + SUM((I,T),SCOSTR(I)*STORER(I,T))
-    + SUM((I,J,T),TCOSTB(I,J)*SHIPB(I,J,T))
-    + SUM((I,T),SCOSTB(I)*STOREB(I,T))

- The price linkages across time are the same as for maize, but for the processed products

Maize Marketing System (cont'd.)

- The FOC for SHIPR(I,J,T) is:
- TCOSTR(I,J) + ROLLER.M(I,T)
- – ROLLER.M(J,T)*(1 – TLOSS) $\geq$ 0
- Or,
- ROLLER.M(I,T) $\geq$ ROLLER.M(J,T)*(1 – TLOSS)
- – TCOSTR(I,J)
- Again, an arbitrage condition

Maize Marketing System (cont'd.)

- The same conditions hold for breakfast where we substitute
  - BFAST.M for ROLLER.M
  - TCOSTB for TCOSTR
- The same is true for the storage variables for the processed products

Maize Marketing System (cont'd.)

- The FOC for STORER(I,T) is
- SCOSTR(I) + ROLLER.M(I,T)
- – ROLLER.M(I,T+1) *(1 – SLOSS)
- – STORER.M(I,T) $\geq$ 0
- or,
- ROLLER.M(I,T) $\geq$ SCOSTR(I)
- \+ ROLLER.M(I,T+1)*(1 – SLOSS) – STORER.M(I,T)

Maize Marketing System (cont'd.)

- and for STOREB(I,T) is
- SCOSTB(I) + BFAST.M(I,T)
- – BFAST.M(I,T+1) *(1 – SLOSS)
- – STOREB.M(I,T) $\geq$ 0
- Or,
- BFAST.M(I,T) $\geq$ SCOSTB(I)
- \+ BFAST.M(I,T+1)*(1 – SLOSS) – STOREB.M(I,T)

Maize Marketing System (cont'd.)

- There is one additional constraint that must be accounted for – processing capacity
- PROCLIM(I,T) ..
- – 1/YIELDR*MAKER(I,T) – 1/YIELDB*MAKEB(I,T)
- =G= – PCAP(I)
- This constraint is stated in tons of maize

Maize Marketing System (cont'd.)

- Now consider the price linkages due to processing
- The FOC for MAKER(I,T) is:
- MCOSTR(I) + MAIZE.M(I,T)*1/YIELDR
- – ROLLER.M(I,T) + PROCLIM.M(I,T)*1/YIELDR
- ≥ 0

Maize Marketing System (cont'd.)

- or,
- ROLLER.M(I,T) $\le$ MAIZE.M(I,T)*1/YIELDR
  - + MCOSTR(I) + PROCLIM.M(I,T)*1/YIELDR
- Again, we have an arbitrage condition

Maize Marketing System (cont'd.)

- The analogous linkages for breakfast processing are:
- The FOC for MAKEB(I,T) :
- MCOSTB(I) + MAIZE.M(I,T)*1/YIELDB
- – BFAST.M(I,T) +PROCLIM.M(I,T)*1/YIELDB
- $\ge$ 0
- or,
- BFAST.M(I,T) $\le$ MAIZE.M(I,T)*1/YIELDB
- + MCOSTB(I) + PROCLIM.M(I,T)*1/YIELDB

Maize Marketing System (cont'd.)

- Note that all of these arbitrage conditions taken together result in a fully linked system of product flows and value:
  - Shipping variables move product across regions and limit price differentials between regions
  - Storage variables move product *forward* through time and limit upward price differentials across periods
  - Processing variables transform raw maize into one of the two products and limit price differentials between forms

Maize Marketing System (cont'd.)

- The forgoing model was complete except for
  - Initial stocks for each of the goods (maize, breakfast, and roller)
  - Outgoing stock requirements of the goods (if you have incoming stocks, it is typical to also have outgoing stock requirements)