1. An important aspect of formulation is the existence of what are called singular points. Singular points are points where a function approaches infinity. So, the function technically does not have a value at the point. For instance \( \ln(0) \) has a singular point at \( x = 0 \). Similarly, \( 1/(x-c) \) has a singular point at \( c \). Consider the following univariate problems with strict inequality constraints.

Maximize \( \ln(x) \)  
Minimize \( 1/(x-c) - px \)  
Subject to: \( px \leq y \)  
\( x > 0 \)  
Subject to: \( x > c \)  

a. For each of these problems, determine the optimum. (It may help you to graph these problems. You should assume that \( p>0, y>0, \) and \( c > 0 \). Check first and second order conditions so that you are certain that you have found an optimum.)

b. Are these convex programs? If so, why? If not, why?

c. In numerical mathematical programming, we cannot use strict inequalities. So the constraints \( x > 0 \) and \( x > c \) in the above problems cannot be used. Rather, we typically use \( x \geq e \) and \( x \geq c + e \) as approximations to these constraints where \( e \) is a very small positive number. Can you always find a number \( e \) that is small enough that the above constraints \( x \geq e \) and \( x \geq c + e \) will not be binding? Why, or why not?

d. Implement and solve the first problem above in GAMS for the values \( p = 7, \) and \( y = 28 \). What happens if you do not set an initial value for \( x \)? Does this happen if you set an initial value of \( x = 20 \)? Is the initial value of 20 feasible?

2. Consider the following convex programming problem:

Minimize \( F(x) \)  
Subject to: \( C_i(x) \geq 0, \ i = 1,2,\ldots,m. \)

a. Let the vector \( x^k \) be the \( k \)-th trial value (iterate) for an iterative process that is meant to solve the above problem. Also, let \( x^k \) be feasible (i.e., I’m telling you it is feasible). Also, let \( x^* \) be the global optimum for this problem. Is there a feasible step from \( x^k \) to \( x^* \)? (By a feasible step, I mean a direction to move from \( x^k \) that is feasible for all values between \( x^k \) to \( x^* \).) Why, or why not?

b. Write down the Lagrangian for the above problem. Is this a convex function of \( x \)? Show why or why not proceeding from the definition of a convex function (i.e., \( L(ax+(1-a)y) \leq aL(x) + (1-a)L(y) \) for \( 0 \leq a \leq 1 \)).